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BINOMIAL COEFFICIENTS WHOSE PRODUCTS ARE PERFECT kTH POWERS

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In memory of Ernst Straus

A P_k-set is a finite set of positions in Pascal's triangle which, when translated anywhere within the triangle, covers entries whose product is a perfect kth power. A characterization of such sets is obtained, and the minimum cardinality $f(k)$ of all P_k-sets is determined.

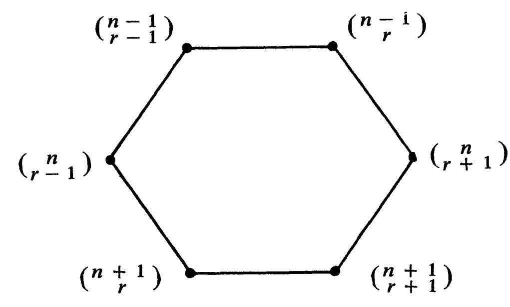
1. Introduction. In 1971, V. Hoggatt and W. Hansell [4] proved that the product of the 6 neighbors of any interior entry of the Pascal triangle is a perfect square. A corollary of this appeared as a problem on the Putnam Examination [9]. In Figure 1, for example, the product of the 6 entries enclosed in circles or squares is $360{,}000 = 600^2$.

The usual proof of this theorem consists in showing that the product of the circled entries is equal to the product of the entries enclosed in squares, i.e.

$$(1)\qquad \binom{n-1}{r-1}\binom{n}{r+1}\binom{n+1}{r} = \binom{n-1}{r}\binom{n}{r-1}\binom{n+1}{r+1}.$$

Because of the positions in Figure 1 of the factors on the 2 sides of (1), this identity has been called the *Star of David property* of Pascal's triangle.

To reformulate the star of David Theorem, we consider the following hexagon:



The theorem says that if this hexagon is translated in such a way that its vertices lie on entries of the Pascal triangle, the product of these entries is