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GENERIC COVERING PROPERTIES FOR SPACES OF ANALYTIC FUNCTIONS

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By a classical result of Fatou, a bounded analytic function on the unit disc D, i.e. in the space $H^\infty(D)$, has a radial limit at almost every point on ∂D. We examine the question of whether this limiting or boundary value lies in the interior or on the boundary of the image domain. We show that the first case is "typical" in the sense that every function in a certain dense G_δ-set of H^∞ has this property at a.e. boundary point. Several other spaces including the disc algebra and the Dirichlet space are also studied.

1. Introduction. Let D be the unit disc in **C**. In this paper we consider certain types of covering properties for analytic functions belonging to the disc algebra $A(D)$, the space of bounded analytic functions $H^\infty(D)$, and the Dirichlet space $\mathscr{D}$. We prove these properties are generic in the categorical sense; i.e., they hold for functions forming a residual set. A generic property is said to hold for "nearly every" function in the space.

For f analytic in d, let E_f denote those points $\zeta \in T = \partial D$ where $f(\zeta)$ fails to exist (as a nontangential limit) or where $f(\zeta)$ exists but $f(\zeta) \notin f(D)$. Our main results are summarized in this

THEOREM. (a) *The Lebesgue measure* $|E_f|$ *is zero for nearly every function* $f \in H^\infty(D)$.

(b) *The logarithmic capacity* Cap(E_f) *is zero for nearly every function* $f \in \mathscr{D}$.

(c) *Let h be any Hausdorff measure function. The Hausdorff measure* $\Lambda_h(E_f)$ *is zero for nearly every function* $f \in A(D)$.

The theorem says that nearly every function in these spaces maps the boundary of the unit disc into the image of the interior; we might say it buries its boundary values. The size of the exceptional set E_f depends upon the space, but our results would seem to be the strongest possible in this regard. One would not expect smaller exceptional sets for $H^\infty(D)$ and $\mathscr{D}$, since nontangential limits may not even exist for sets of measure zero and capacity zero, respectively. This doesn't happen in $A(D)$, since these functions are continuous on $\bar{D}$; however, we have shown that by almost any measure of smallness, nearly every function in $A(D)$ has a "small"