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UNIVERSAL OBSERVABILITY AND CODIMENSION ONE SUBGROUPS OF BOREL SUBGROUPS

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A subgroup H of an affine algebraic group G is *observable in* G if the quotient variety G/H is quasi-affine (equivalently, if each character on H is the character of a one-dimensional H-submodule of an irreducible G-module). The question is how to characterize the universally observable groups, i.e., those which are observable in every group in which they can be embedded. We remark that the relation that G/H should be *affine* for every embedding of H is equivalent to H being reductive, by work of Cline, Parshall, Scott. A sufficient condition for the universal observability of a solvable group is that a certain monoid of characters for the inner operation of H on its hyperalgebra should be a group. Here, we give a two-dimensional example (a codimension one subgroup of a Borel subgroup of GL_2) to show that this sufficient condition is not necessary. Secondly, we give a method for testing a group for the *failure* of universal observability, which we use to show the *non*-universal observability of a famly of codimension one subgroups of Borel subgroups of GL_n $(n \geq 3)$. We remark that the universal observability of an affine algebraic group is equivalent to the universal observability of its solvable radical. Consequently, we only need to sort the solvable groups for those that are universally observable.

Introduction. An affine algebraic group H is called *universally observable* if every embedding of H in an affine algebraic group G gives a quasi-affine quotient G/H (cf. [**1**]). Here we study aspects of universal observability for solvable groups which are codimension one subgroups of Borel subgroups of Gl_n or Sl_n.

We use two methods to establish the universal observability of certain groups. (1) Sweedler's method: The inner operation of H on its hyperalgebra generates a monoid of characters (see §1). That this monoid should be a group is a sufficient condition for universal observability of a solvable group. (2) Direct calculation for two-dimensonal groups for which the first method fails (see §2). This work began as a study of whether the sufficient condition (1) for universal observability was also necessary. Our principal results are as follows. (1) Using the first method, we show that $\mathrm{Ker}\,\chi$ is universally observable when $\chi\colon B \to G_m$ is a character in the dominant chamber of a Borel subgroup B of Sl_n. (2) In characteristic zero, we show the universal observability of the semidirect product $G_a \times_w G_m$, where the