PACIFIC JOURNAL OF MATHEMATICS
Vol. 122, No 2, 1986

A NOTE ON THE G-SPACE VERSION OF GLICKSBERG'S THEOREM

J. DE VRIES

In an earlier paper, the author generalized Glicksberg's theorem about the Stone-Čech compactifiction of products to the context of G-spaces and their maximal G-compactifications, where G is an arbitrary locally compact group, acting on all spaces under consideration. However, in that paper only products of finitely many factors were considered. In the present note, infinite products are taken into account.

A note on the G-space version of Glicksberg's theorem. This note is a supplement to [2] and [3]. In [2] the theorem below was proved for finite products (with "G-pseudocompact" instead of "pseudocompact"). Later in [3] it was shown that G-pseudocompactness is equivalent to pseudocompactness. Using the result from [3], we are now able to prove the theorem in its full generality. For notation and terminology we refer to [2]. In particular, G is a locally compact topological group and all G-spaces have completely regular Hausdorff phase spaces.

THEOREM. *Let $\{\langle X_\lambda, \pi_\lambda\rangle\colon \lambda \in \Lambda\}$ be a set of G-spaces. Then the following statements hold true*:

(i) *Suppose G is locally connected and there exists a partition $\Lambda = \Lambda \cup \Delta$ such that both $\prod_{\gamma \in \Gamma} X_\gamma$ and $\prod_{\delta \in \Delta} X_\delta$ are G-infinite. If $\beta_G(\prod_{\lambda \in \Lambda} X_\lambda) = \prod_{\lambda \in \Lambda} \beta_G X_\lambda$, then $\prod_{\lambda \in \Lambda} X_\lambda$ is pseudocompact.*

(ii) *If $\prod_{\lambda \in \Lambda} X_\lambda$ is pseudocompact, then $\beta_G(\prod_{\lambda \in \Lambda} X_\lambda) = \prod_{\lambda \in \Lambda} \beta_G X_\lambda$.*

Proof. (i) In [2] this statement was proved for a product of two factors (note, that by [3] the conclusion of [2] that the product is G-pseudocompact, implies that the product is pseudocompact). So we have to reduce the case of infinite products to the case of a product of two factors. This can be done exactly as in [1], once the following claim has been proved:

Claim. If $\beta_G(\prod_{\lambda \in \Lambda} X_\lambda) = \prod_{\lambda \in \Lambda} \beta_G X_\lambda$, then for every subset Γ of Λ one has $\beta_G(\prod_{\gamma \in \Gamma} X_\gamma) = \prod_{\gamma \in \Gamma} \beta_G X_\gamma$. (This claim holds true without the additional conditions, mentioned in the theorem above.) The proof of this claim cannot be given similar as in (one of the footnotes of) [1], because in general the embedding of a subproduct in the full product cannot be