PACIFIC JOURNAL OF MATHEMATICS
Vol 123, No. 1, 1986

# DERIVATIONS WITH INVERTIBLE VALUES IN RINGS WITH INVOLUTION

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**Let $R$ be a semiprime 2-torsion free ring with involution * and let $S = \{x \in R \mid x = x^*\}$ be the set of symmetric elements. We prove that if $R$ has a derivation $d$, non-zero on $S$, such that for all $s \in S$ either $d(s) = 0$ or $d(s)$ is invertible, then $R$ must be one of the following: (1) a division ring, (2) $2 \times 2$ matrices over a division ring, (3) the direct sum of a division ring and its opposite with exchange involution, (4) the direct sum of $2 \times 2$ matrices over a division ring and its opposite with exchange involution, (5) $4 \times 4$ matrices over a field with symplectic involution.**

Recently Bergen, Herstein and Lanski studied the structure of a ring $R$ with a derivation $d \neq 0$ such that, for each $x \in R$, $d(x) = 0$ or $d(x)$ is invertible. They proved that, except for a special case which occurs when $2R = 0$, such a ring must be either a division ring $D$ or the ring $D_2$ of $2 \times 2$ matrices over a division ring.

In this paper we address ourselves to a similar problem in the setting of rings with involution, namely: let $R$ be a 2-torsion free semiprime ring with involution and let $S$ be the set of symmetric elements. If $d \neq 0$ is a derivation of $R$ such that the non-zero elements of $d(S)$ are invertible, what can we conclude about $R$?

We shall prove that $R$ must be rather special. In fact we shall show the following:

THEOREM. *Let $R$ be a 2-torsion free semiprime ring with involution. Let $d$ be a derivation of $R$ such that $d(S) \neq 0$ and the non-zero elements of $d(S)$ are invertible in $R$. Then $R$ is either*:

1. *a division ring $D$, or*
2. *$D_2$, the ring of $2 \times 2$ matrices over $D$, or*
3. *$D \oplus D^{\mathrm{op}}$, the direct sum of a division ring and its opposite relative to the exchange involution, or*
4. *$D_2 \oplus D_2^{\mathrm{op}}$ with the exchange involution, or*
5. *$F_4$, the ring of $4 \times 4$ matrices over a field $F$ with symplectic involution.*

In case $R = F_4$ with * symplectic we shall prove that $d$ is inner. As Herstein has pointed out, an easy example of such a ring is given by