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SPECTRAL DECOMPOSITION OF $L^2(N \setminus \mathrm{GL}(2), \eta)$

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Let G denote $\mathrm{GL}(2, F)$, where F is $\mathbf{R}$ or $\mathbf{C}$ or a p-adic field. Let η be a non-trivial character of the unipotent upper triangular group N in G. The object of this paper is to present an explicit spectral decomposition of $L^2(N \setminus G, \eta)$, the representation of G induced unitarily by η. This representation is well-known to be multiplicity-free and to be quasi-equivalent to the (right) regular representation of G.

Introduction. In §1 we give a 'cuspidal' characterization of the discrete spectrum of $L^2(N \setminus \mathrm{SL}(2, F), \eta)$. In §2 we prove a crucial duality formula, which later allows us to decompose the continuous spectrum. It also 'explains' the occurrence of ε'-factors in the measure giving the direct integral decomposition.

The scalar product of §3 is originally due, in the p-adic case, to H. Jacquet. Here we suitably modify and extend his (unpublished) work and also treat the archimedean case. We have tried to present a unified approach.

This work was done as a part of my doctoral thesis [**11**] at Columbia. Besides the obvious debt to my advisor then, H. Jacquet, I would like to mention the strong influence of R. Godement's paper ([**3**]) on the spectral analysis of modular functions, and thank P. Sally for his interest and critical comments. I would like to thank the referee for his helpful remarks which led to simplifications of some of the proofs. Thanks are also due to Miss M. Murray and Mrs. Anne Wolfsheimer for their excellent typing of this manuscript.

0. Terminology. Fix a non-trivial character ψ of F^+. Let dx be the self-dual measure on F with respect to ψ and let $|\ \ |$ be the normalized absolute value. When F is non-archimedean, let $\mathfrak{O}$, $\mathfrak{p}$, v, π and $\mathbf{F}_q$ respectively denote the ring of integers, maximal ideal, valuation, uniformizer and residue field. On F^2 we will always take the product measure induced by the self-dual measure dx on F. We will denote by U_F the group $\{x \in F \mid |x| = 1\}$.