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ON FUNCTIONS AND STRATIFIABLE μ-SPACES

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It is shown that a space X is a stratifiable μ-space if and only if X has a topology induced by the collection $\bigcup_{n=1}^{\infty} \Phi_n$ of $[0,1]$-valued continuous functions of X such that each Φ_n satisfies the conditions (α), (β) and (γ) stated below.

1. Introduction. Throughout, all spaces are assumed to be regular Hausdorff. N always denotes the positive integers. For a space X, $C(X, I)$ denotes the collection of all continuous functions $f\colon X \to I = [0,1]$. For $f \in C(X, I)$ we denote by $\operatorname{coz} f$ the cozero set of f in X. We are assumed to be familiar with the class of stratifiable spaces in the sense of [**1**]. For a stratifiable space X, every closed subset F of X has a stratification $\{O_n(F) : n \in N\}$ in X. As is well-known, every stratifiable space X is monotonically normal, that is, X has a monotonically normal operator $D(M, N)$ for each disjoint pair (M, N) of closed subsets of X.

J. Guthrie and M. Henry characterized metrizable spaces X in terms of collections of continuous functions with continuous sup and inf as follows: A space X is metrizable if and only if X has the weak topology induced by a σ-relatively complete collection $\Phi \subset C(X, I)$, that is, $\Phi = \bigcup_{n=1}^{\infty} \Phi_n$, where for each n, each subcollection of Φ_n has both continuous sup and inf, [**3**]. On the other hand, C. R. Borges and G. Gruenhage obtained the characterization of stratifiable spaces as follows: A space X is stratifiable if and only if for each open set U of X there exists $f_U \in C(X, I)$ such that $\operatorname{coz} f_U = U$ and such that for each family $\mathscr{U}$ of open subsets of X, $\sup\{f_U : U \in \mathscr{U}\} \in C(X, I)$, [**2**, Theorem 2.1]. In the discussion below, we also give a characterization of the class of stratifiable μ-spaces in terms of collections of continuous functions with continuous sups with an additional condition. This is the main purpose of this paper.

In an earlier paper [**6**], the author introduced the notion of M-structures and studied the class $\mathscr{M}$ of all stratifiable spaces having an M-structure. This class $\mathscr{M}$ is shown to coincide with that of stratifiable μ-spaces, [**5**]. The kernel of M-structures is the term "$\mathscr{H}$-preserving in both sides". Therefore, first we state the definition. For the definition of M-structures, we refer the reader to [**6**].

Let $\mathscr{U}$, $\mathscr{H}$ be families of subsets of a space X. Then we call that $\mathscr{U}$ is *inside* $\mathscr{H}$-preserving at a point $p \in X$ if for each $\mathscr{U}_0 \subset \mathscr{U}$, $p \in \bigcap \mathscr{U}_0$