PACIFIC JOURNAL OF MATHEMATICS
Vol. 125, No. 2, 1986

UNITARY COLLIGATIONS IN Π_κ-SPACES, CHARACTERISTIC FUNCTIONS AND ŠTRAUS EXTENSIONS

A. DIJKSMA, H. LANGER AND H. S. V. DE SNOO

Dedicated to Earl A. Coddington on the occasion of his 65th birthday.

The main result of this paper is the description of certain linear manifolds $T(\lambda)$, associated with a symmetric operator, in terms of certain boundary values of the characteristic function of a unitary colligation.

1. Introduction. Let $\mathfrak{H}$ be a Hilbert space and let $\tilde{\mathfrak{H}}$ be a π_k-space, i.e., a Pontryagin space with κ negative squares, such that $\tilde{\mathfrak{H}}$ contains $\mathfrak{H}$ and the indefinite inner product on $\tilde{\mathfrak{H}}$ restricted to $\mathfrak{H}$ coincides with the Hilbert inner product on $\mathfrak{H}$; we denote this situation by $\mathfrak{H} \subset_s \tilde{\mathfrak{H}}$. Let A be a selfadjoint subspace in $\tilde{\mathfrak{H}}^2$ with nonempty resolvent set $\rho(A)$. With A we associate as in **[8]** a family $\{T(l) \mid l \in \mathbf{C} \cup \{\infty\}\}$ of linear manifolds $T(l) \subset \mathfrak{H}^2$ defined by

$$(1.1)\quad \begin{cases} T(l) = \{\{Pf, Pg\} \mid \{f, g\} \in A, g - lf \in \mathfrak{H}\}, & l \in \mathbf{C}, \\ T(\infty) = \{\{f, Pg\} \mid \{f, g\} \in A, f \in \mathfrak{H}\}. \end{cases}$$

Here P denotes the orthogonal projection from $\tilde{\mathfrak{H}}$ onto $\mathfrak{H}$. We note that $A \cap \mathfrak{H}^2$ is a symmetric subspace in $\mathfrak{H}^2$, with adjoint $(P^{(2)}A)^c$, i.e., the closure in $\mathfrak{H}^2$ of the set

$$P^{(2)}A = \{\{Pf, Pg\} \mid \{f, g\} \in A\}.$$

The following inclusions are obvious:

$$A \cap \mathfrak{H}^2 \subset T(l) \subset P^{(2)}A, \qquad l \in \mathbf{C} \cup \{\infty\},$$

and also

$$T(\bar{l}) \subset T(l)^*, \qquad l \in \mathbf{C} \cup \{\infty\},$$

with equality when $l \in \rho(A)$.

Now let S be a symmetric subspace in $\mathfrak{H}^2$. We consider the selfadjoint extensions $A \subset \tilde{\mathfrak{H}}^2$ of S, with nonempty resolvent set $\rho(A)$, where $\mathfrak{H} \subset_s \tilde{\mathfrak{H}}$. The corresponding families $\{T(l) \mid l \in \mathbf{C} \cup \{\infty\}\}$, form the class of Štraus extensions of S and $T(l)$ for $l \in \mathbf{C} \setminus \mathbf{R}$ was characterized in **[8]**, to which