PACIFIC JOURNAL OF MATHEMATICS
Vol. 127, No. 1, 1987

ROTATION NUMBERS FOR AUTOMORPHISMS OF C^* ALGEBRAS

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Poincaré's notion of rotation number for a homeomorphism of the circle is generalized to a large class of automorphisms of C^* algebras. This is accomplished by the introduction of a C^* algebraic notion of determinant. A formula is obtained for the range of a trace on the K_0 group of a cross product by Z in terms of the rotation number of the automorphism involved.

Introduction. In [16] Poincaré introduced the notion of rotation number for homeomorphisms of the circle. The idea is to associate to any orientation-preserving homeomorphism of the circle a complex number of absolute value one which, in some sense, represents the average amount by which each individual point in the circle is "rotated" by the given homeomorphism. If R_θ denotes the rotation by the angle θ on the circle, that is, the transformation $z \to e^{i\theta}z$, we may compute its rotation number which, not surprisingly, turns out to be equal to $e^{i\theta}$.

Suppose we replace the circle by the 2-torus T^2 (viewed as the cartesian product of two circles) and let $R_{\eta,\theta}$ be the homeomorphism of T^2 which rotates the first and second circle coordinates by different angles η and θ. It seems plausible to assert that $R_{\eta,\theta}$ admits two rotation numbers, namely $e^{i\eta}$ and $e^{i\theta}$.