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LOCALIZATION IN THE CLASSIFICATION OF FLAT MANIFOLDS

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Two compact flat Riemannian manifolds are called comparable if each one is a covering space of the other in such a way that the covering maps are affine and both the compositions of the covering maps increase distance locally by a constant factor. Considering comparability classes instead of affine-equivalence classes corresponds to localizing the algebra in calculations.

Introduction. This paper is concerned with compact flat Riemannian manifolds, i.e. smooth compact manifolds with a Riemannian connection for which the Levi-Civita connection is flat. These are all quotients of Euclidean space $\mathbf{R}^n$ by a group of isometries Γ acting properly discontinuously. A continuous map between two such manifolds is called affine if it lifts to an affine map of $\mathbf{R}^n$. The rotational part of Γ, i.e. its image in $\mathrm{GL}(\mathbf{R}^n)$, is called the holonomy group of the manifold and is always finite. Charlap [**4**] showed that the affine-equivalence classes of these manifolds with given holonomy group G correspond bijectively with the isomorphism classes of a category $E_{\mathbf{Z}}(G)$ defined in terms of the integral representations of G. For the purpose of calculation it is convenient to localize the integral representations to get a category $\hat{E}(G)$. This will be seen to correspond to the following geometric notion.

DEFINITION. Two compact flat Riemannian manifolds B_1, B_2 are comparable if there exist affine covering maps θ_1: $B_1 \to B_2$, θ_2: $B_2 \to B_1$ such that $\theta_1 \circ \theta_2$ and $\theta_2 \circ \theta_1$ both increase distance locally by a factor m.

Section 1 covers the background material. In §2 we shall look at the endomorphisms of these manifolds and in §3 we shall prove the following.

THEOREM A. *There is a natural bijection between the isomorphism classes of $\hat{E}(G)$ and the comparability classes of compact flat Riemannian manifolds with holonomy group G.*