CONNECTEDNESS RELATED TO ALMOST PERIODICITY OF COMPOSITIONS OF FLOW HOMOMORPHISMS

DOUG C. MCMAHON, JAAP VAN DER WOUDE AND TA-SUN WU

Consider homomorphisms ϕ: $X \to Y$ and ψ: $Y \to Z$, where ϕ is open and N-to-one, ψ is almost periodic. In the paper by R. J Sacker and G. R. Sell, it was shown that, under a certain condition on the phase group, the composition $\psi \circ \phi$: $X \to Z$ is almost periodic (provided that Z is trivial and X is minimal). In this paper almost periodicity of $\psi \circ \phi$ is studied under connectedness conditions on the fibers of ψ. For instance it is shown that if ψ is almost periodic with connected fibers than $\psi \cdot \phi$ is almost periodic. If ψ is locally almost periodic with locally connected fibers then $\phi \circ \psi$ is locally almost periodic.

0. Introduction. The results in this paper contribute to a list of achievements with respect to the question, when is the composition of almost periodic extensions again almost periodic? In general such a composition is not almost periodic (viz: the existence of distal non-almost periodic flows). But under certain connectedness conditions on the maps involved ([**E 69**] page 56, [**W 75**]) and/or under a certain assumption for the phase group ([**SS 74**], [**MW 76**], [**B 75 / 79**], [**R ?**]) some compositions are.

The impetus for this research came from [**SS 74**], where it was shown an open N-to-one extension of an almost periodic flow was an almost periodic flow again as long as the phase group was semicompactly generated (i.e., there is a compact $K \subseteq T$ such that every open $V \supseteq K$ generates T). In [**MW 76**] this result was strengthened to the relativized nonmetric case, while a connection was made with connectedness conditions on the fibers. This resulted in the theorem that, for a semicompactly generated phase group, minimal distal flows with 0-dimensional phase space are almost periodic. In [**W 75**] it was shown, without conditions on the phase group, that an open N-to-one extension of a connected almost periodic regular minima flow was almost periodic. Various results along this line are obtained in [**B 75 / 79**] and [**R ?**] with respect to manifold flows and Lie group extensions; but the conditions imposed on the phase groups and phase spaces are rather strong, mostly due to the perspective of applications.