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A CHARACTERIZATION THEOREM FOR COMPACT UNIONS OF TWO STARSHAPED SETS IN R^3

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Set S in R^d has property P_k if and only if S is a finite union of d-polytopes and for every finite set F in bdry$S$ there exist points $c_1, \dots, c_k$ (depending on F) such that each point of F is clearly visible via S from at least one c_i, $1 \leq i \leq k$. The following results are established.

(1) Let $S \subseteq R^3$. If S satisfies property P_2, then S is a union of two starshaped sets.

(2) Let $S \subseteq R^d$, $d \geq 3$. If S is a compact union of k starshaped sets, then there exists a sequence $\{S_j\}$ converging to S (relative to the Hausdorff metric) such that each set S_j satisfies property P_k.

When $d = 3$ and $k = 2$, the converse of (2) above holds as well, yielding a characterization theorem for compact unions of two starshaped sets in R^3.

1. Introduction. We begin with some definitions. Let S be a subset of R^d. Hyperplane H is said to *support S locally* at boundary point s of S if and only if $s \in H$ and there is some neighborhood N of s such that $N \cap S$ lies in one of the closed halfspaces determined by H. Point s in S is called a *point of local convexity* of S if and only if there is some neighborhood N of s such that $N \cap S$ is convex. If S fails to be locally convex at q in S, then q is called a *point of local nonconvexity* (lnc point) of S. For points x and y in S, we say x *sees* y via S (x is *visible* from y via S) if and only if the segment $[x, y]$ lies in S. Similarly, x is *clearly visible* from y via S if and only if there is some neighborhood N of x such that y sees via S each point of $N \cap S$. Set S is *locally starshaped* at point x of S if and only if there is some neighborhood N of x such that x sees via S each point of $N \cap S$. Finally, set S is *starshaped* if and only if there is some point p in S such that p sees via S each point of S, and the set of all such points p is called the (convex) *kernel* of S.

A well-known theorem of Krasnosel'skii [3] states that if S is a nonempty compact set in R^d, S is starshaped if and only if every $d + 1$ points of S are visible via S from a common point. Moreover, "points of S" may be replaced by "boundary points of S" to produce a stronger result. In [1], the concept of clear visibility, together with work by Lawrence, Hare, and Kenelly [4], were used to obtain the following