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PIECEWISE LINEAR FIBRATIONS

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By an ANR *fibration* we will mean a Hurewicz fibration $p: E \to B$, where E is a compact ANR and B is a compact polyhedron. In case E is also a polyhedron and p is a piecewise linear (PL) map, we say that E is a PL fibration. An important special case of this is the notion of a PL manifold bundle, which is a PL locally trivial bundle for which the fibers are compact PL manifolds (with boundary). It is known that any ANR fibration $E \to B$ is "homotopic" to a PL manifold bundle $\mathscr{E} \to B$ in the sense that there exists a path through ANR fibrations from E to $\mathscr{E}$. This takes the form of an ANR fibration over $B \times [0,1]$ whose 0-level is E and whose 1-level is $\mathscr{E}$. The purpose of this paper is to prove that if E is additionally assumed to be a PL fibration, then the ANR fibration over $B \times [0,1]$ can be chosen to be a PL fibration.

This establishes a PL link between the categories {PL fibrations} and {PL manifold bundles}, and it is hoped that this will provide a convenient framework for applying the methods of algebraic K-theory to the study of PL manifold bundles. This was the strategy that was adopted in [**8**], but unfortunately there are gaps in the argument. In particular the appropriate PL link was not established. Our Theorem 3, which is stated below, does establish this PL link. Its proof relies on Theorem 1, which is a PL local connectivity result for spaces of PL maps having contractible point-inverses. The main tool used in establishing Theorem 1 is a stable version of the Fibered Controlled h-Cobordism Theorem of [**5**].

In order to state Theorem 1 we will have to introduce some notation. If X and Y are compact polyhedra, then a PL surjection $r: X \to Y$ is said to be a *contractible map* (c-map) if all of the point-inverses are contractible. A *c-homotopy* r_t: $X \to Y$ is a fiber-preserving (f.p.) c-map $r = \{r_t\}$: $X \times [0,1] \to Y \times [0,1]$. When we say that a statement is *stably* true regarding X, we actually mean that there is an integer k for which the corresponding statement is true for $X \times I^k$, where I^k is the k-cell $[0,1]^k$. Similarly when we say that a statement is stably true regarding a map r: $X \to Y$, we actually mean that there is an integer k for which the corresponding statement is true for the composition $X \times I^k \xrightarrow{\text{proj}} X \xrightarrow{r} Y$. The first result that we establish is the following local connectivity result. In its statement we use Δ^n for the standard n-simplex.