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FOUR DIMENSIONAL HOMOGENEOUS ALGEBRAS

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An algebra is homogeneous if the automorphism group acts transitively on the one dimensional subspaces of the algebra. The purpose of this paper is to determine all homogeneous algebras of dimension 4. It continues previous work of the authors in which all homogeneous algebras of dimensions 2 and 3 were described. Our main result is the proof that the field must be $GF(2)$ and the algebras are of a type previously described by Kostrikin. There are 5 non-isomorphic algebras of dimension 4; a description of each is given and the automorphism group is calculated in each case.

All algebras considered are finite dimensional and not necessarily associative. By $\mathrm{Aut}(A)$ we denote the group of algebra automorphisms of the algebra A. Thus, an algebra A is homogeneous if $\mathrm{Aut}(A)$ acts transitively on the one dimensional subspaces of A. A general discussion of homogeneous algebras may be found in [**8**] along with references to related literature. Djokovic [**2**] has classified all homogeneous algebras over the field of real numbers and has found that the only examples exist in dimensions 3, 6 and 7. Sweet [**9**] has shown that non-trivial examples cannot exist over any algebraically closed field. Homogeneous algebras of dimension 2 were studied in [**8**] where it was shown the field must be $GF(2)$. The authors have also previously classified dimension 3 homogeneous algebras [**6**] where it was found that either the algebra is a truncated quaternion algebra or else the field must be $GF(2)$. The purpose of this paper is to determine the structure and automorphism group of all homogeneous algebras of dimension 4.

Kostrikin has shown, in [**5**], how to construct homogeneous algebras over the field $GF(2)$ in every dimension.

DEFINITION. *Let $K = GF(2^n)$ and let μ be any fixed element in K. Let $\circ\colon K \times K \to K$ be the map defined by $x \circ y = \mu(xy)^{2^{n-1}}$. Then $A(n, \mu)$ denotes the algebra over $GF(2)$ obtained by replacing the usual multiplication in K by the map $\circ$. We call $A(n, \mu)$ a Kostrikin Algebra.*

These algebras are shown to be homogeneous by Kostrikin and are obviously commutative. We can now state the main result of the paper. It is summarized in the following theorem.