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EIGENFUNCTIONS OF THE NONLINEAR EQUATION $\Delta u + \nu f(x, u) = 0$ IN R^2

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In this paper we consider the existence of eigenfunctions of the boundary value problem for the nonlinear equation mentioned in the title with vanishing boundary values on bounded planar domains.

Let Ω be a bounded domain in R^2. In this paper we consider the existence of eigenfunctions of the boundary value problem

$$(0.1) \qquad \begin{cases} \Delta u + \nu f(x, u) = 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where f is a continuous function in both x and u variables for all $(x, u) \in \Omega \times R$. We assume that f satisfies the growth condition

$$(0.2) \qquad \begin{cases} f(x, 0) = 0 \\ |f(x, u)| \leq A + B|u|^m e^{\alpha u^2} \text{ uniformly in } x \end{cases}$$

for some nonnegative constants A, B, m and $\alpha > 0$. We note that $u \equiv 0$ is a trivial solution for (0.1). Let $H_0^1(\Omega)$ denote the completion of the space of compactly supported C^1 functions on Ω under the norm

$$\|u\|_{H_0^1(\Omega)} = \left(\int_\Omega |\nabla u|^2\right)^{1/2}.$$

We set $F(x, u) = \int_0^u f(x, s)\, ds$. Our main results are the following:

THEOREM 1. *Let $f(x, u)$ be a continuous function in $(x, u) \in \Omega \times R$ and f satisfies condition* (0.2). *For any $\mu > 0$ such that there exists a $v \in H_0^1(\Omega)$ with $\int_\Omega |\nabla v|^2 = \gamma < (4\pi/\alpha)$ and $\int_\Omega F(x, v) = \mu$, the eigenvalue problem* (0.1) *has a nontrivial eigenfunction u satisfying $\int_\Omega F(x, u) = \mu$.*

If we are interested in positive solutions, a similar theorem applies.

THEOREM 2. *Let $f(x, u)$ be a continuous function in $(x, u) \in \Omega \times R$ that satisfies condition* (0.2) *and the condition*

$$(0.2') \qquad f(x, u) > 0 \quad \text{if } u > 0.$$