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THE DIVISOR FUNCTION AT CONSECUTIVE INTEGERS

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Let $d(n)$ be the divisor function. Improving a result of Heath-Brown, we show that for sufficiently large x, $d(n) = d(n+1)$ holds for $\gg x(\log\log x)^{-3}$ integers $n \le x$.

1. Introduction. In a recent paper with the above title, Heath-Brown [5] showed that there are infinitely many positive integers n, for which $d(n) = d(n+1)$. In fact, he proved that for sufficiently large x

$$\#\{n \le x: d(n) = d(n+1)\} \gg x(\log x)^{-7}. \tag{1.1}$$

This settled a problem of Erdös and Mirsky [1]. Heath-Brown's work was motivated by an earlier result of Spiro [6], namely that with $a = 5040$, $d(n) = d(n+a)$ holds for infinitely many integers n.

Using a different approach, Erdös, Pomerance and Sarközy [2] recently showed that for sufficiently large x

$$\sum_{i=0}^{3} \#\{n \le x: d(n) = 2^i d(n+1)\} \gg x(\log\log x)^{-1/2}, \tag{1.2}$$

and in a subsequent paper [3] established the upper bound

$$\#\{n \le x: d(n) = d(n+1)\} \ll x(\log\log x)^{-1/2}. \tag{1.3}$$

These results strongly suggest that the right order of magnitude for the quantity estimated by (1.1) and (1.3) is $x(\log\log x)^{-1/2}$. A heuristic argument supporting this conjecture has been given P. T. Bateman and C. Spiro.

The main purpose of this paper is to prove the following estimate, which considerably improves on Heath-Brown's bound (1.1) and falls short of the conjectured bound only by a power of $\log\log x$.

THEOREM 1. *For sufficiently large x,*

$$\#\{n \le x: d(n) = d(n+1)\} \gg x(\log\log x)^{-3}. \tag{1.4}$$

The idea of the proof is to combine the methods of Heath-Brown and Erdös-Pomerance-Sarközy. We shall give an outline of the proof in §3.