PACIFIC JOURNAL OF MATHEMATICS
Vol. 130, No. 2, 1987

ON THE GEOMETRY OF EXTENSIONS OF IRREDUCIBLE MODULES FOR SIMPLE ALGEBRAIC GROUPS

STEPHEN R. DOTY AND JOHN B. SULLIVAN

Let G be a simple, simply connected affine algebraic group over an algebraically closed field k of non-zero characteristic p. We consider the problem of determining the extensions of irreducible modules by irreducible modules. The extensions may be realized as submodules of modules induced from characters on a Borel subgroup of G. The geometry of the distribution of composition factors of those induced modules is determined by an operation (namely, alcove transition) of the Weyl group on the space of weights. Generically in the lowest p^2-alcove, that operation stabilizes a canonical subset of the set of highest weights of those irreducible modules which extend the irreducible module of some fixed highest weight. The stability leads to an upper bound on that subset, which can be refined using the translation principle. We give a conjecture for the generic distribution of extensions of irreducible modules by a fixed irreducible module.

Introduction. Let G be a simple, simply connected affine algebraic group over an algebraically closed field k of non-zero characteristic p. (B,T) is a fixed Borel subgroup and maximal torus pair, $X(B)$ is the character group of B, and B^{opp} is the opposite Borel subgroup. Take the positive roots of (G,T) to be the roots of (B^{opp},T). Let G_1 be the kernel of the Frobenius morphism of G. Let $\{H^i(\chi)\}_{i=0}^{\dim G/B} = \{H^i(G/B, L(\chi))\}_{i=0}^{\dim G/B}$ be the sheaf cohomology modules of the homogeneous space G/B at the line bundle $L(\chi)$ induced from a character χ on B.

For each dominant character λ, $H^0(\lambda)$ has as its socle the irreducible module M_λ of highest weight λ. The formal character of M_λ can be computed in terms of the formal characters of the modules $\{M_\mu \mid \mu \neq \lambda,\ \mu$ strongly linked to $\lambda\}$, once the multiplicities $[H^0(\lambda)\colon M_\mu]$ of the M_μ as composition factors of $H^0(\lambda)$ are known. Let $X[H^0(\lambda)] = \{(\mu, n) \in X(B) \times \mathbf{Z}_{\geq 0} \mid [H^0(\lambda)\colon M_\mu] = n\}$, let $X_\lambda = \{\mu \in X(B) \mid [H^0(\lambda)\colon M_\mu] \neq 0\}$, and let $X^\lambda = \{\mu \in X(B) \mid [H^0(\mu)\colon M_\lambda] \neq 0\}$. In [7], the authors defined the W-linkage class $WL \cdot \lambda$ of a character λ. They showed that $WL \cdot \lambda$ is an upper bound for X_λ, when λ is a weight generic in the