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ROTATION SETS OF MAPS OF THE ANNULUS

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We study how the rotation interval of a point affects the rotation set of its ω-limit set. Similarities between rotation set and topological entropy were found, suggesting a deeper relation between these two concepts.

Introduction. In 1913 Birkhoff proved the first important theorem about twist maps of the annulus, initially conjectured by Poincaré and known as "*Poincaré's last geometric theorem*". It roughly says that any area preserving homeomorphism of the annulus which rotates the boundary components in opposite directions has at least two fixed points.

In **[6]** John Franks, using an extension of Poincaré's definition of rotation number, proved that any chain transitive homeomorphism of the annulus that twists the boundaries in opposite directions has at least one fixed point; it follows from his work that if a homeomorphism of the annulus preserves area then either it has infinitely many periods or every point has the same rotation number.

In this paper we introduce the rotation set for an endomorphism of the annulus. Associated to each orbit we define a real sequence that measures the "wrapping" of the orbit around the inner boundary of the annulus. The limit of this sequence, if it exists, is called the *rotation number* of the orbit. If the limit does not exist, the set of all limit points of the referred sequence is a closed interval and designated *rotation interval* (see pg. 253). The rotation set is the union of all rotation intervals. It is a topologically invariant set that contains its supremum and infimum; however we don't know if it is always closed.

There are interesting similarities between properties of the *rotation set* and *topological entropy*. Katok in **[11]** proved that any $C^{1+\varepsilon}$ diffeomorphism, f, with positive entropy has an invariant hyperbolic set Λ where f is topologically conjugate to a shift. This implies that either every point in Λ has the same rotation number or there is some point without rotation number. Nevertheless the relation between *positive entropy* and *existence of points without rotation number* is not completely understood.