PACIFIC JOURNAL OF MATHEMATICS
Vol. 136, No. 2, 1989

LOCALIZATION OF A CERTAIN SUBGROUP OF SELF-HOMOTOPY EQUIVALENCES

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Let X be a simple, finite C.W. complex. The group $\mathscr{E}_\#(X)$ is known to be nilpotent. In this paper, we give a proof of the naturality of localization on this group, $\mathscr{E}_\#(X)_{(P)} = \mathscr{E}_\#(X_{(P)})$. The result is then applied to study the group structures of $\mathscr{E}_\#(X)$ of rational Hopf spaces and some Lie groups.

Introduction. Let X be a pointed topological space. We use the notation $\mathscr{E}(X)$ to denote the group of based self-homotopy equivalences of X. (For this group there are other notations, for example, $\mathrm{AUT}^\circ(X)$ in [**2**].) Throughout the paper our spaces X will be connected of finite type and either finite dimensional or Postnikov pieces, namely, spaces with finite number of non-trivial homotopy groups. Then we denote by $\mathscr{E}_\#^m(X)$ the subgroup of $\mathscr{E}(X)$ which is the kernel of the obvious map (cf. [**1**], [**16**]):

$$\mathscr{E}(X) \to \prod_{j\le m} \operatorname{Aut} \pi_j(X).$$

We simply denote $\mathscr{E}_\#(X)$ when $m = \dim X$, where

$$\dim X = \max\{i \mid \pi_i(X) \ne 0\}$$

if X is a Postnikov piece.

E. Dror and A. Zabrodsky have proved that $\mathscr{E}_\#(X)$ is a nilpotent group for an arbitrary finite dimensional C.W. complex or a Postnikov piece ([**2**], Theorem A). If $m \ge \dim X$, $\mathscr{E}_\#^m(X)$ is a subgroup of $\mathscr{E}_\#(X)$ and thus also nilpotent. Hence these groups can be localized in a natural way. For example, the reader may consult the book [**5**] which provides basic matters on the theory of localization of nilpotent groups (and spaces).

In this paper, our main result is the following.

THEOREM 0.1. *Let X be a simple C.W. complex and P be an arbitrary collection of prime numbers. Assume that $m \ge \dim X$. Then the*