PACIFIC JOURNAL OF MATHEMATICS
Vol. 137, No. 2, 1989

THE INJECTIVE FACTORS OF TYPE III_λ, $0 < \lambda < 1$

UFFE HAAGERUP

Dedicated to the memory of Henry A. Dye

We give a new proof for Connes' result that an injective factor of type III_λ, $0 < \lambda < 1$ on a separable Hilbert space is isomorphic to the Powers factor R_λ. Our approach is based on lengthy, but relatively simple operations with completely positive maps together with a technical result that gives a necessary condition for that two n-tuples $(\xi_1, \dots, \xi_n)$ and $(\eta_1, \dots, \eta_n)$ of unit vectors in a Hilbert W^*-bimodule are almost unitary equivalent. As a step in the proof we obtain the following strong version of Dixmier's approximation theorem for III_λ-factors: Let N be a factor of type III_λ, $0 < \lambda < 1$, and let φ be a normal faithful state on N for which $\sigma_{t_0}^\varphi = \mathrm{id}$ ($t_0 = -2\pi/\log\lambda$); then for every $x \in N$ the norm closure of $\mathrm{conv}\{uxu^* | u \in U(M_\varphi)\}$ contains a scalar operator.

1. Introduction and preliminaries. In [**6**, §7] Connes proved that, for each $\lambda \in \,]0,1[$, there is up to isomorphism only one injective factor of type III_λ (with separable predual), namely the Powers factor,

$$R_\lambda = \bigotimes_{n=1}^{\infty}(M_2, \varphi_\lambda).$$

Here M_2 is the algebra of complex 2×2-matrices and φ_λ is the state on M_2 given by

$$\varphi_\lambda\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22}\end{pmatrix} = \frac{1}{1+\lambda}(\lambda\varphi(x_{11}) + \varphi(x_{22})).$$

(The notion R_λ was introduced by Araki and Woods in [**1**]. In Powers' original work [**19**], R_λ denoted M_α, where $\alpha = \lambda/(1+\lambda)$.)

Connes' approach for proving uniqueness of the injective factor of type III_λ ($\lambda \in \,]0,1[$ fixed) is the following: By [**4**, §4] every factor N of type III_λ has an essentially unique crossed product decomposition

$$N = P \times_\theta Z$$

where P is a II_∞-factor and θ is an isomorphism of P for which $\tau\circ\theta = \lambda\tau$, where τ is a normal faithful semifinite trace on P. Moreover, N is