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HYPERFINITE VON NEUMANN ALGEBRAS AND POISSON BOUNDARIES OF TIME DEPENDENT RANDOM WALKS

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Dedicated to the memory of Henry A. Dye

We consider the problem of characterizing Poisson boundaries of group-invariant time-dependent Markov random walks on locally compact groups G. We show that such Poisson boundaries, which by construction are naturally G-spaces, are amenable and approximately transitive (see Definition 1.1 and Theorem 2.2).

We also establish a relationship between von Neumann algebras and Poisson boundaries when $G = R$ or Z. More precisely, there is naturally associated to an eigenvalue list for an ITPFI factor M, a group-invariant time-dependent Markov random walk on R whose Poisson boundary is the flow of weights for M (Theorem 3.1).

0. Introduction. Henry Dye's work has a lasting impact on ergodic theory and operator algebras. We present this paper, which deals with both of these subjects, as a tribute to his mathematical achievements and his gentle and unassuming character.

We consider the problem of characterizing Poisson boundaries of group-invariant time-dependent Markov random walks on locally compact groups G. We show that such Poisson boundaries, which by construction are naturally G-spaces, are amenable and approximately transitive (see Definition 1.1 and Theorem 2.2). We believe that the converse also holds, namely that these two conditions precisely characterize such Poisson boundaries. Under the pressure of time, we have not yet completed our proof. However it is true in the transitive case (Theorem 2.4), and when $G = R$ or Z (Theorems 3.2 and 3.4). Theorem 2.6 is the beginning of our attack on the general case.

We also establish a relationship between von Neumann algebras and Poisson boundaries when $G = R$ or Z. More precisely, there is naturally associated to an eigenvalue list for an ITPFI factor M, a group-invariant time-dependent Markov random walk on R whose Poisson boundary is the flow of weights for M (Theorem 3.1). Theorem 3.3 gives the corresponding result for $G = Z$. This unexpected identification has interesting applications in both directions. Using