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PROPAGATION OF HYPO-ANALYTICITY ALONG BICHARACTERISTICS

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It is shown here the hypo-analytic singularities for solutions propagate along the bicharacteristics of hypo-analytic differential operators of principal type. This generalizes the well-known similar result for analytic differential operators.

0. Introduction. In [4] Hormander proved a result concerning the propagation of C^∞ singularities of solutions of $Pu = f$ for a smooth linear partial differential operator P whose leading symbol is real. The analytic version of this question was treated by Hanges in [3]. In this paper we prove a similar theorem for what we call hypo-analytic differential operators. The paper is organized as follows. In §1 we discuss the structures we work in and introduce our operators. In §2 we recall the definition of microlocal hypo-analyticity and give a statement of the main result. §3 discusses the Fourier transform criterion of microlocal hypo-analyticity due to Baouendi, Chang and Treves [1]. A theorem concerning this criterion is proved in the same section and then used in the proof of our main result.

1. Hypo-analytic differential operators. Our results deal with structures which are a special case of the hypo-analytic structures introduced in [1]. Let Ω be a C^∞ manifold of dimension m. A hypo-analytic structure of maximal dimension on Ω is the data of an open covering (U_α) of Ω and for each index α, of m C^∞ functions $Z_\alpha^1, \dots, Z_\alpha^m$ satisfying the following two conditions:

(i) $dZ_\alpha^1, \dots, dZ_\alpha^m$ are linearly independent at each point of U_α;

(ii) if $U_\alpha \cap U_\beta \neq \varnothing$, there are open neighbors O_α of $Z_\alpha(U_\alpha \cap U_\beta)$ and O_β of $Z_\beta(U_\alpha \cap U_\beta)$ and a holomorphic map F_β^α of O_α onto O_β, such that

$$Z_\beta = F_\beta^\alpha \circ Z_\alpha \quad \text{on } U_\alpha \cap U_\beta.$$

We will use the notation $Z_\alpha = (Z_\alpha^1, \dots, Z_\alpha^m)\colon U_\alpha \to C^m$. A distribution h defined on an open neighborhood of a point p_0 of Ω is called hypo-analytic at p_0 if there is a local chart (U_α, Z_α) of the above type whose domain contains p_0 and a holomorphic function $\tilde{h}$ defined on