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EQUIVARIANT COMPLETELY BOUNDED OPERATORS

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An equivariant completely bounded linear operator between two C^*-algebras acted on by an amenable group is shown to lift to a completely bounded operator between the crossed products that is equivariant with respect to the dual coactions. A similar result is proved for coactions and dual actions. It is shown that the only equivariant linear operators that lift twice through the action and dual coaction of an infinite group are the completely bounded ones.

1. Introduction. Let $\Phi: A \to B$ be a bounded linear map between C^*-algebras, and let $\alpha: G \to \operatorname{Aut} A$ and $\beta: G \to \operatorname{Aut} B$ be actions of an amenable locally compact group G. If Φ is a homomorphism and is equivariant—i.e., $\Phi(\alpha_s(a)) = \beta_s(\Phi(a))$ for $s \in G$, $a \in A$—then it extends to a homomorphism $\Phi \times i$ from the crossed product $A \rtimes_\alpha G$ to $B \rtimes_\beta G$. On the other hand, if C is another C^*-algebra, then for any completely bounded map $\Phi: A \to B$ there is a bounded operator $\Phi \times i: A \otimes C \to B \otimes C$; indeed, by taking C to be the algebra $\mathscr{K}$ of compact operators, we can see that the complete boundedness of Φ is necessary for this to be true. We shall combine these results to prove that any equivariant completely bounded operator extends to a bounded map on the crossed product, formulate and prove the analogous results for crossed products by coactions of nonabelian groups, and investigate the extent to which complete boundedness is a necessary hypothesis.

We shall take similar approaches to the problems of lifting through actions and coactions. First, we prove equivariant versions of Stinespring's theorem [23] on completely positive maps into $\mathscr{B}(\mathscr{H})$, and we then show how to modify Wittstock's theorem [25] to write an equivariant symmetric completely bounded map into $\mathscr{B}(\mathscr{H})$ as a difference of equivariant completely positive ones. From these we can use standard symmetrisation techniques to deduce that an equivariant completely bounded operator $\Phi: A \to \mathscr{B}(\mathscr{H})$ can be realized in the form $\Phi(a) = T\pi(a)V$, where π is part of a covariant representation (π, U) of (A, G, α) in some larger Hilbert space. We can then define $\Phi \times i: A \rtimes_\alpha G \to \mathscr{B}(\mathscr{H})$ by the formula $\Phi \times i(z) = T(\pi \times U(z))V$, and the result for $\Phi: A \to B$ follows easily.