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# NONRATIONAL FIXED FIELDS

JAMES K. DEVENEY AND JOE YANIK

**We present an example of a flag of rational extensions, stabilized by the action of the group of order 2 such that the fixed field under the group action is not retract rational and hence not rational. This fixed field is shown to be a genus 0 extension of a pure transcendental extension.**

Let $L \supset K \supset F$ be fields finitely generated over $F$. If $K = F(X_1, \dots, X_n)$ where $\{X_1, \dots, X_n\}$ is algebraically independent over $F$ then $K$ is a rational extension of $F$. Saltman defined $K$ to be a *retract rational extension of* $F$ if $K$ is the quotient field of an $F$-algebra $A$ and there are maps $f: F[X_1, \dots, X_n](1/w) \to A$ and $g: A \to F[X_1, \dots, X_n](1/w)$ such that $f \circ g = \mathrm{id}$, where $\{X_1, \dots, X_n\}$ is algebraically independent over $F$ and $w \in F[X_1, \dots, X_n]$. If rational extensions are considered free objects then retract rational extensions could in some sense be considered as projective objects.

Let $G$ be a finite group of $k$-automorphisms of a rational function field $k(X_1, \dots, X_n)$. Assume that the "flag" of subfields $\{k[X_1, \dots, X_i)/1 \leq i \leq n\}$ is stablized by $G$. Then in many situations, for example if $|G|$ is odd, the fixed field of $G$ will be rational over $k$ [**10**, Lemma 4, p. 322]. We present an example of $G$ as above, where $|G| = 2$ and the fixed field of $G$ is not even retract rational. We also describe this field as a genus 0 extension of a pure transcendental extension of the rational members.

Let $\alpha$ be the automorphism of $Q(X_1, X_2, X_3, X_4, Z_1, \dots, Z_8)$ defined by

$$\begin{aligned} \alpha(X_i) &= X_{i+1} \quad \text{for } 1 \leq i \leq 3, \\ &\;\; - X_1 \quad \text{for } i = 4, \\ \alpha(Z_i) &= Z_{i+1} \quad \text{for } 1 \leq i \leq 7, \\ &\;\;\;\; Z_1 \quad \text{for } i = 8. \end{aligned}$$

Then $\alpha$ is a $k$-automorphism of order 8 and induces a $G$-action on $Q(X_1, X_2, X_3, X_4, Z_1, \dots, Z_8)$ where $G = C_8$. Furthermore, the restriction of $\alpha$ induces a faithful $G$-action on each of $Q(X_1, X_2, X_3, X_4)$ and $Q(Z_1, \dots, Z_8)$. By [**2**, Propositioon 1.4, p. 303] this implies that $Q(X_1, X_2, X_3, X_4, Z_1, \dots, Z_8)^\alpha$ is a rational extension of both