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ON THE EIGENVALUES OF REPRESENTATIONS OF REFLECTION GROUPS AND WREATH PRODUCTS

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Let G be a finite group. The eigenvalues of any $g \in G$ of order m in a (complex) representation ρ may be expressed in the form $\omega^{e_1}, \omega^{e_2}, \ldots$, with $\omega = e^{2\pi i/m}$. We call the integers $e_j \pmod{m}$ the cyclic exponents of g with respect to ρ. We give explicit combinatorial descriptions of the cyclic exponents of the (irreducible) representations of the symmetric groups, the classical Weyl groups, and certain finite unitary reflection groups. We also show that for any finite group G, the cyclic exponents of the wreath product $G \wr S_n$ can be described in terms of the cyclic exponents of G. For each of the infinite families of finite unitary reflection groups W, we also provide explicit, combinatorial descriptions of the generalized exponents of W. These parameters arise in the symmetric algebra of the associated reflection representation, and by a theorem of Springer, are closely related to the cyclic exponents of W.

0. Introduction. Let G be a finite group acting on a complex vector space V. If $g \in G$ is an element of order m, then the eigenvalues of g on V are mth roots of unity, and may therefore be expressed in the form $\omega^{e_1}, \omega^{e_2}, \ldots$, with $\omega = e^{2\pi i/m}$. We call the integers e_j (mod m) the *cyclic exponents* of g with respect to V. This terminology is partly inspired by the case in which G is a Weyl group and V carries the reflection representation of G. If $g \in G$ is a Coxeter element, then the corresponding cyclic exponents e_j, reduced mod m to the form $0 \leq e_j < m$, are the classical exponents of G [**B**].

The central objective of this paper is to provide explicit descriptions of cyclic exponents for groups G whose irreducible representations have intrinsic combinatorial structure. In the typical situation, we have a group G acting (irreducibly) on some space V whose dimension is in one-to-one correspondence with a set C of combinatorial objects. (The prototype we have in mind is the case in which $G = S_n$ and C is a set of standard Young tableaux.) The problem to be solved is to find, for each $g \in G$, a "natural" rule for attaching an exponent $e = e_g(c)$ to each of the combinatorial objects $c \in C$, so that $\{\omega^{e_g(c)} \colon c \in C\}$ is the multiset of eigenvalues of g on V.