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THE WAVE FRONT SET AND THE ASYMPTOTIC SUPPORT FOR p-ADIC GROUPS

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We prove that for p-adic groups the notion of the wave front set of a representation coincides with the notion of the asymptotic support.

1. The wave front sets of finite sums of homogeneous distributions. Let Ω be a p-adic field of characteristic zero, with valuation $|\cdot|$. Let $\mathbf{g}$ be a finite dimensional vector space over Ω. Fix a non-trivial character χ of the additive group Ω, and a non-degenerate symmetric bilinear form β on $\mathbf{g}$ with values in Ω.

For $f \in C_c^\infty(\mathbf{g})$ (compactly supported, locally constant functions on $\mathbf{g}$) define a Fourier Transform by

$$\hat{f}(Y) = \int_{\mathbf{g}} \chi(\beta(Y, X)) f(X)\, dX \qquad (Y \in \mathbf{g}). \tag{1.1}$$

Here dX is a Haar measure on the additive group of $\mathbf{g}$ (normalized so that the formula $(\hat{f})^\wedge(x) = f(-x)$ holds). Then $f \to \hat{f}$ is a bijective mapping of $C_c^\infty(\mathbf{g})$ onto itself (see **[Ha1]** or [**W**, p. 107]). If T is a distribution $\mathbf{g}$ then its Fourier transform $\hat{T}$ is given by

$$\hat{T}(f) = T(\hat{f}) \qquad (f \in C_c^\infty(\mathbf{g})). \tag{1.2}$$

Let $n = \dim_\Omega(\mathbf{g})$. For $f \in C_c^\infty(\mathbf{g})$ define

$$f_\lambda(X) = |\lambda|^{-n} f(\lambda^{-1} X) \qquad (X \in \mathbf{g},\ \lambda \in \Omega^\times). \tag{1.3}$$

Fix an open subgroup Λ of Ω^x with $[\Omega^x : \Lambda] < \infty$.

DEFINITION 1.4. A distribution T on $\mathbf{g}$ is Λ-homogeneous of degree $d \in \mathbf{C}$ if

$$T(f_\lambda) = |\lambda|^d T(f) \qquad (f \in C_c^\infty(\mathbf{g}),\ \lambda \in \Lambda).$$

Notice that

$$(f_\lambda)^\wedge = |\lambda|^{-n} (\hat{f})_{\lambda^{-1}} \qquad (f \in C_c^\infty(\mathbf{g}),\ \lambda \in \Omega^x), \tag{1.5}$$

so that if T is Λ-homogeneous of degree d then $\hat{T}$ is a Λ-homogeneous of degree $-n-d$. Clearly if T is a function:

$$T(f) = \int_{\mathbf{g}} T(X) f(X)\, dX,$$