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CONFORMAL CLUSTER SETS AND BOUNDARY CLUSTER SETS COINCIDE

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The result stated in the title is proved. No restriction other than the obvious requirement that the cluster sets be taken at a nonisolated boundary point is imposed on the domain where the mapping is defined. The result is then generalized by allowing for certain exceptional sets on the boundary. More refined versions are established in the special case where the domain is the open unit disk. These include the statement that one-sided cluster sets coincide with one-sided radial cluster sets. Again, certain exceptional sets on the boundary are allowed for. Consequences are presented in which the existence of limits along sets on the boundary implies limits inside the domain. Finally, generalizations to the class of homeomorphisms satisfying the Carathéodory Prime End Theorem are indicated.

1. Introduction. In this paper we show that

$$C(f,b)=C_{\partial D}(f,b) \tag{1}$$

for a conformal mapping f of a domain D in the extended complex plane $\overline{\mathbb{C}}$. Here $C(f,b)$ denotes the cluster set of f at a boundary point b of D, while $C_{\partial D}(f,b)$ denotes the boundary cluster set of f at b (see §2 for definitions). We impose no restrictions on the domain D other than the requirement that the boundary point b be non-isolated, which is essential if $C_{\partial D}(f,b)$ is to be nonempty. Thus D can be of finite or infinite connectivity. In §3 relation (1) is generalized to

$$C(f,b)=C_{\partial D-E}(f,b) \tag{2}$$

allowing for certain exceptional sets E on ∂D. For instance, a condition guaranteeing (2) will be given in terms of logarithmic measure.

The much studied case where D is the open unit disk B will be considered next. Several extensions of the classical theorem of Iversen and Tsuji [**4**, p. 91] will be presented in the conformal case. For instance, we establish (2), with $D=B$, for any set E on ∂B satisfying one of the following six conditions:

1°. E is of capacity zero.
2°. E is of linear measure zero.
3°. E is an O_{AD}-set.