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ON COMPLETE SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS IN BANACH SPACES

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This paper is concerned with the complete second order equation $u''(t)+Bu'(t)+Au(t)=0$ in a Banach space, where both A and B are densely defined closed linear operators. The main result is a theorem of Hill-Yosida-Phillips type for the Cauchy problem for the equation to be well posed.

1. Introduction and the main result. Consider the complete second order linear differential equation

$$u''(t)+Bu'(t)+Au(t)=0 \qquad (t\geq 0) \tag{1.1}$$

in a complete Banach space E, where A, B are densely defined closed linear operators. The equation has been extensively studied by semi-group methods during the last thirty years. A great amount of literature on it can be looked up in Fattorini's monograph [**1**] which was published in 1985. However, as stated in [**1**, Ch. VIII], the theory of (1.1) "can hardly be said in definitive form".

Let us begin with the restatements of some definitions in [**1**]:

DEFINITION 1. We say that an E-valued function $u(t)$ defined in $t\geq 0$ is a solution of (1.1) if $u(t)$ is twice continuously differentiable, $u(t)\in D(A)$, $u'(t)\in D(B)$, $Au(t)$ and $Bu'(t)$ are continuous and (1.1) is satisfied in $t\geq 0$.

DEFINITION 2. We say that the Cauchy problem for (1.1) is well posed if the following two assumptions are satisfied:

(a) There exist dense subspaces D_0, D_1 of E such that, for any $u_0\in D_0$, $u_1\in D_1$, there exists a solution $u(t)$ of (1.1) with $u(0)=u_0$, $u'(0)=u_1$.

(b) There exists a nondecreasing, nonnegative function $N(t)$ defined in $t\geq 0$ such that

$$\|u(t)\|\leq N(t)(\|u(0)\|+\|u'(0)\|) \qquad (t\geq 0) \tag{1.2}$$

for any solution of (1.1).