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SOLVABILITY OF INVARIANT DIFFERENTIAL OPERATORS ON METABELIAN GROUPS

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In this work we use non-commutative harmonic analysis in the study of differential operators on a certain class of solvable Lie groups. A left invariant differential (a differential operator that commutes with left translations on the group) can be synthesized in terms of differential operators on lower dimensional spaces. This synthesis is easily described for a certain class of simply connected solvable Lie groups, those arising as semi-direct products of simply connected abelian groups.

We derive sufficient conditions for the semiglobal solvability of left invariant differential operators on such groups in terms of the lower dimensional differential operators. These conditions are seen to be satisfied for certain classes of second order differential operators, thus yielding semiglobal solvability. Specifically elliptic, sub-elliptic, transversally elliptic and parabolic operators are investigated.

1. Introduction. In the mid 1970's the study of differential operators with polynomial coefficients that arise as invariant operators on nilpotent Lie groups began. Group representation theoretic criteria for hypo-ellipticity and solvability of such operators were found. Rothschild [**11**] gave such criteria for the local solvability of homogeneous left invariant differential operators on the Heisenberg group based on some ideas of Rockland [**10**]. These ideas were extended to all simply connected nilpotent Lie groups by Corwin in [**1**]. In recent years these ideas and methods have been extended to type I solvable Lie groups. Lipsman [**6**] gave criteria for local solvability of left invariant differential operators on type I solvable Lie groups, thus extending the horizon of study to differential operators with more general (transcendental) coefficients.

In the work done so far on one sided invariant operators, non-commutative harmonic analysis plays an important role. Specifically the Plancherel theory is an essential ingredient. In this work we demonstrate an alternative approach, one which doesn't use the Plancherel theory. One advantage is that we obtain solvability results for operators on non-type I groups for which there is no reasonable Plancherel theory. In addition our approach is successful in obtaining