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THE MOD 2 EQUIVARIANT COHOMOLOGY ALGEBRAS OF CONFIGURATION SPACES

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Dedicated to my mother

The mod 2 equivariant cohomology algebras of configuration spaces are determined by means of the Dickson characteristic classes, which are derived from the modular invariants of the general linear groups $\mathrm{GL}(n, \mathbb{Z}_2)$ and closely related to the Stiefel-Whitney classes.

Introduction. Let us consider the configuration space $F(\mathbb{R}^q, m) = \{(x_1, \dots, x_m): x_i \in \mathbb{R}^q, x_i \neq x_j \text{ if } i \neq j, 1 \leq i, j \leq m\}$ as a free $\mathfrak{S}_m$-space, where the action of the symmetric group $\mathfrak{S}_m$ of degree m is given by permutations of the factors. As is well known, the limit of the spaces of orbits

$$F(\mathbb{R}^\infty, m)/\mathfrak{S}_m = \varinjlim_q F(\mathbb{R}^q, m)/\mathfrak{S}_m$$

becomes a classifying space of $\mathfrak{S}_m$. Meanwhile the limit

$$H^*(F(\mathbb{R}^q, \infty)/\mathfrak{S}_\infty; \mathbb{Z}_p) = \varprojlim_m H^*(F(\mathbb{R}^q, m)/\mathfrak{S}_m; \mathbb{Z}_p)$$

is equipped with the Hopf algebra structure introduced essentially by M. Nakaoka [**12**; §2] for $1 \leq q \leq \infty$. Here $\mathbb{Z}_p$ denotes the prime field of p elements.

Let

$$J_{\text{odd}}(q) = \{(h_0, \dots, h_{n-1}) \neq 0; n > 0, h_i \in \mathbb{Z}_+, h_0 + \cdots + h_{n-1} < q, \text{ there exists } j \text{ such that } h_j \text{ is odd}\}$$

for $1 \leq q \leq \infty$. Then, in [**13**], [**15**] for each $H = (h_0, \dots, h_{n-1}) \in J_{\text{odd}}(\infty)$ we have introduced the universal Dickson characteristic class

$$W^H \in H^*(\mathfrak{S}_\infty; \mathbb{Z}_2) = \varprojlim_m H^*(\mathfrak{S}_m; \mathbb{Z}_2)$$

of degree $\dim(W^H) = \sum_{s=0}^{n-1} h_s(2^n - 2^s)$. Here we use the name of L. E. Dickson because these classes are related to his modular invariant theory as seen in [**15**]. We have proved the following.