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RELATIONS AMONG GENERALIZED CHARACTERISTIC CLASSES

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In this paper, we extend Brown and Peterson's algebraic calculations, using methods of homotopy theory, to the consideration of manifolds with structure and to characteristic classes arising from generalized cohomology theories.

0. Introduction. In [**BP**], E. Brown and F. Peterson made the first calculation of relations among the Stiefel-Whitney classes of the stable normal bundles of manifolds. Specifically, they computed

$$I_n = \bigcap_{M^n} \operatorname{Ker} \nu_M^* \subset H^*(BO; \mathbb{Z}/2),$$

where $\nu_M \colon M^n \to BO$ classifies the stable normal bundle of M^n, and the intersection is taken over all compact differentiable manifolds of dimension n. These calculations have, via the Brown-Gitler spectra [**BG**], proven to be of considerable value. Although they arose in the context of the Immersion Conjecture for compact differentiable manifolds and were instrumental in its solution [**C1**], these spectra were also used by M. Mahowald [**Ma**], and subsequently at odd primes by R. Cohen [**C2**], to produce infinite families in the homotopy groups of spheres. G. Carlsson used the Spanier-Whitehead duals of these spectra to prove the Segal Conjecture for elementary abelian 2-groups [**Ca**], and H. Miller then used the algebra thus developed by Carlsson in his proof of the Sullivan Conjecture [**Mi**].

These theories should be related to the bordism theories coming from our chosen class of manifolds. We wish to calculate

$$I_n = \bigcap_{(M^n, \tilde{\nu})} \operatorname{Ker} \tilde{\nu}^* \subset E^*(B),$$

where B is the classifying space associated to a certain class of manifolds, denoted by pairs $(M^n, \tilde{\nu})$; $\tilde{\nu} \colon M^n \to B$ is a lifting of ν_M (B comes equipped with a map to BO); and E^* is the cohomology theory. We will place the following conditions on E^*, where TB is the Thom-spectrum associated to B.