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SEQUENCE TRANSFORMATIONS THAT GUARANTEE A GIVEN RATE OF CONVERGENCE

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Let t be a positive number sequence and define the sequence space $\Omega(t) := \{x : x_k = O(t_k)\}$. Characterizations are given for matrices that map the spaces l^1, l^∞, c, or c_0 into $\Omega(t)$, thus ensuring that the transformed sequence converges at least as fast as t. These results yield information about matrices that map l^1, l^∞, c, or c_0 into $G := \bigcup_{r\in(0,1)} \Omega(r^n)$, the set of geometrically dominated sequences.

1. Introduction. For each r in the interval $(0, 1)$ let

$$G(r) = \{\text{complex sequences } x : x_k = O(r^k)\}$$

and define the set of *geometrically dominated sequences* as

$$G = \bigcup_{r\in(0,1)} G(r).$$

The *analytic sequences* are defined by

$$\mathscr{A} = \left\{\text{complex sequences } x : \limsup_n |x_n|^{1/n} < \infty\right\}.$$

Obviously $G \subseteq \mathscr{A}$. In **[2, 6, 9, 10]** the various authors studied matrix transformations from $\mathscr{A}$ or G into l^1, c, or l^∞, but the question of mapping *from* l^1, c, or l^∞ *into* $\mathscr{A}$ or G was not considered. We shall use the customary notation for a matrix transformation: if A is an infinite matrix with complex entries and x is a complex number sequence, then A transforms x into the sequence Ax whose nth term is given by

$$(Ax)_n = \sum_{k=0}^{\infty} a_{nk}x_k.$$

The present work began as a study of $l^1 - G$ and $c - G$ matrices, but it was found that such results are merely special cases of a more general theory. To set the stage for the general theory we replace the geometric sequence $\{r^k\}$ with a nonnegative sequence t and define

$$\Omega(t) = \{x : x_k = O(t_k)\}.$$