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MANIFOLD SUBGROUPS OF THE HOMEOMORPHISM GROUP OF A COMPACT Q-MANIFOLD

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Let X be a compact PL manifold and Q denote the Hilbert cube I^ω. In this paper, we show that the following subgroups of the homeomorphism group $H(X \times Q)$ of $X \times Q$ are manifolds:

$$H^{\mathrm{fd}}(X \times Q) = \{h \times \mathrm{id} \mid h \in H(X \times I^n) \text{ for some } n \in \mathbb{N}\},$$
$$H^{\mathrm{PL}}(X \times Q) = \{h \times \mathrm{id} \in H^{\mathrm{fd}}(X \times Q) \mid h \text{ is } PL\} \text{ and}$$
$$H^{\mathrm{LIP}}(X \times Q) = \text{all Lipschitz homomorphisms of } X \times Q \text{ under some suitably chosen metric}.$$

In fact, let $\widetilde{H}^*(X \times Q)$ denote the subspace consisting of those homeomorphisms which are isotopic to a member of $H^*(X \times Q)$, where $* = \mathrm{fd}$, PL or LIP respectively. Then it is shown that

(1) $(\widetilde{H}^{\mathrm{PL}}(X \times Q), H^{\mathrm{PL}}(X \times Q))$ is an (l_2, l_2^f)-manifold pair,

(2) $(\widetilde{H}^{\mathrm{LIP}}(X \times Q), H^{\mathrm{LIP}}(X \times Q))$ is an (l_2, l_2^Q)-manifold pair and

(3) $H^{\mathrm{fd}}(X \times Q)$ is an $(l_2 \times l_2^f)$-manifold and dense in $\widetilde{H}^{\mathrm{fd}}(X \times Q)$, where l_2 is the separable Hilbert space, $l_2^f = \{(x_i) \in l_2 \mid x_i = 0$ except for finitely many $i\}$ and $l_2^Q = \{(x_i) \in l_2 \mid \sup |i \cdot x_i| < \infty\}$.

0. Introduction. By $H(X)$, we denote the homeomorphism group of a compactum X onto itself with the compact-open topology. Let $Q = I^\omega$ be the Hilbert cube and l_2 the separable Hilbert space. A separable manifold modeled on Q or l_2 is called a *Q-manifold* or *l_2-manifold*, respectively. By the combined works of **[Ge$_1$]**, **[To$_1$]** and **[Fe]** or **[To$_2$]**, it was shown that $H(M)$ is an l_2-manifold for a compact Q-manifold M. In this paper, we concern ourselves with subgroups of $H(M)$ which are manifolds.

Let l_2^f and l_2^Q be the linear spans of the natural orthonormal basis of l_2 and the Hilbert cube $\prod_{n\in\mathbb{N}}[-1/n, 1/n]$ in l_2, respectively, that is

$$l_2^f = \{x \in l_2 \mid x(n) = 0 \text{ except for finitely many } n\},$$
$$l_2^Q = \left\{x \in l_2 \mid \sup_{n\in\mathbb{N}} |n \cdot x(n)| < \infty\right\},$$

where $x(n)$ denotes the nth coordinate of x. A separable manifold