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EXPLICIT $\overline{\partial}$-PRIMITIVES OF HENKIN-LEITERER KERNELS ON STEIN MANIFOLDS

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In this paper we construct explicitly $\overline{\partial}$-primitives and use them to obtain a representation formula for holomorphic functions and a theorem on extendability of CR-functions.

1. Introduction. Let X be a Stein manifold of dimension n, $h : X \to \mathbb{C}^p$ $(p \leq n-1)$ a holomorphic map and let $Z(h) = \{\zeta \in X : h(\zeta) = 0\}$. If $K(\zeta, z) = K^{(s^*, \nu)}(\zeta, z)$ is a Henkin-Leiterer type kernel on X (see §2 for notation) then $K(\zeta, z)$ is a $\overline{\partial}$-closed $(n, n-1)$-form in ζ, for a fixed z, i.e., $\overline{\partial}_\zeta K(\zeta, z) = 0$, whose singularity occurs at $\zeta = z$. On the other hand, since $X - Z(h)$ is $(n-2)$-complete (see Sorani and Villani [8, p. 435]), it follows that the cohomology group

$$H^{n-1}(X - Z(h), \mathscr{O}^n) \cong H_{\overline{\partial}}^{(n, n-1)}(X - Z(h))$$

vanishes (see Andreotti and Grauert [1, p. 250]). Therefore, for a fixed $z \in Z(h)$, there exists an $(n, n-2)$-form $\eta(\zeta, z)$, in $X - Z(h)$, so that

$$\overline{\partial}_\zeta \eta(\zeta, z) = K(\zeta, z).$$

For some problems, however, it is important to have explicit formulas for such $\overline{\partial}$-primitives, η, of K; the problems we have in mind are related to integral representations (see for example Stout [9] and Hatziafratis [2]) and extendability of CR-functions (see for example Lupacciolu [6], Tomassini [11] and Stout [10]). Since such forms $\eta(\zeta, z)$ are not unique, their dependence on z, for example, may be difficult to control with cohomological arguments.

In this paper we construct explicitly such $\overline{\partial}$-primitives and use them to obtain a representation formula for holomorphic functions and a theorem on extendability of CR-functions.

The arrangement of the paper is as follows. First in §2 we review the main points of the Henkin-Leiterer construction; with X and h as above we consider a domain $D \subset X$, a Stein neighborhood W of $\overline{D}$ and we briefly discuss what a Leray section $s^* = s^*(\zeta, z)$ and the associated Henkin-Leiterer kernel $K(\zeta, z) = K^{(s^*, \nu)}(\zeta, z)$ are.