PACIFIC JOURNAL OF MATHEMATICS
Vol. 150, No. 1, 1991

TRACE RINGS FOR VERBALLY PRIME ALGEBRAS

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Every algebra p.i. equivalent to some $M_{k,l}$ and with zero annihilator of the Razmyslov ideal has a central extension with non-degenerate trace.

A p.i. algebra R is said to be *verbally prime* if whenever $f(x_1, \dots, x_n)g(x_{n+1}, \dots, x_m)$ is an identity for R then either f or g is. Kemer introduced these algebras in [3]. Since then, verbally prime algebras have been the subject of a number of papers. At the end of this paper we will summarize the results known to us. A common theme is that verbally prime algebras have many properties in common with prime p.i. algebras. In the present work we continue this thread and show that certain verbally prime algebras have "nice" embeddings into trace rings.

Kemer proved that every non-trivial verbally prime p.i. algebra in characteristic zero must be p.i. equivalent to either: $n \times n$ matrices over the field; or $n \times n$ matrices over an infinite dimensional Grassmann algebra E; or $M_{k,l}$. The algebra $M_{k,l}$ is a certain subalgebra of the $(k+l)\times(k+l)$-matrices over E. E has a natural $Z/2Z$-grading $E = E_0 \oplus E_1$, in which E_0 is spanned by the even words and E_1 is spanned by the odd words. Then $M_{k,l}$ consists of all $(k+l)\times(k+l)$ matrices of the form $\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)$, where A is a $k \times k$-matrix with entries in E_0, D is an $l \times l$-matrix with entries E_0, and B and C have entries in E_1.

The algebra $M_{k,l}$ has a trace function $\mathrm{tr}\colon M_{k,l} \to E_0$ defined by $\mathrm{tr}(\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)) = \mathrm{tr}\, A - \mathrm{tr}\, D$. This function satisfies all of the usual properties of the usual properties of trace: it takes values in E_0, the center of $M_{k,l}$; it is E_0-linear; and $\mathrm{tr}(xy) = \mathrm{tr}(yx)$ for all $x, y \in M_{k,l}$. Razmyslov [5] studied the central polynomials and trace identities of $M_{k,l}$. He found non-vanishing, multilinear central polynomials $p(x_1, \dots, x_n, a)$ and $c(x_1, \dots, x_n)$ for $M_{k,l}$ with the property that

$$(*) \qquad p(x_1, \dots, x_n, a) = c(x_1, \dots, x_n)\mathrm{tr}(a)$$

on $M_{k,l}$. For convenience, we generally abbreviate $p(x_1, \dots, x_n, a)$ to $p(x, a)$ and $c(x_1, \dots, x_n)$ to $c(x)$.