PACIFIC JOURNAL OF MATHEMATICS
Vol. 151, No. 1, 1991

ON CENTRAL TYPE FACTOR GROUPS

SHAHRIAR SHAHRIARI

Γ is a group of central type if it possesses an irreducible complex character of degree $|\Gamma: \mathbf{Z}(\Gamma)|^{1/2}$. This is the largest possible degree for an ordinary irreducible character of a finite group. A group G which is isomorphic to $\Gamma/\mathbf{Z}(\Gamma)$, where Γ is some group of central type, is called a central type factor group (ctfg). A variety of restrictions on ctfgs are found. These include a local characterization of ctfgs, and restrictions on normal and subnormal structures of ctfgs.

1. Introduction. It is easy to see (see Corollary 2.30 of **[10]**) that the degree of an irreducible character of a finite group Γ cannot be larger than $|\Gamma: \mathbf{Z}(\Gamma)|^{1/2}$. A group Γ that has an irreducible character of this maximal degree is called a group of central type. A group G which is isomorphic to $\Gamma/\mathbf{Z}(\Gamma)$, where Γ is some group of central type, is called a central type factor group (ctfg for short).

A configuration that occurs often in character theory of solvable groups, and therefore has been the object of much research, is that of fully ramified sections (see **[4]**, **[6]**, **[8]**, **[9]**, **[14]**). A normal subgroup N of Γ is said to be fully ramified in Γ, if there exists an irreducible character θ of N, such that $\theta^{\Gamma} = e\chi$, for χ some irreducible character of G, and $\chi_N = e\theta$. Now, if N is fully ramified in Γ, then Γ/N is a ctfg (see Lemma 2.6), and thus the study of fully ramified sections reduces to that of ctfgs. Another reason why characterizations of ctfgs are sought after is that a group G is a ctfg if and only if it possesses a 2-cocycle α such that the twisted group algebra $\mathbb{C}^{\alpha}[G]$ is simple (see Theorem 2.7).

The study of these groups goes back to Iwahori and Matsumoto in 1964. They conjectured that ctfgs must be solvable (see **[11]**). Various properties of ctfgs were discovered in **[3]**, **[4]**, and **[12]**. In 1982, Howlett and Isaacs **[6]** proved the solvability of ctfgs.

The problem of understanding solvable ctfgs remains open. In this paper, we will get more restrictions on the structure of a ctfg.

DeMeyer and Janusz **[3]** proved that if G is a ctfg then so are its Sylow subgroups. However, to get that G is a ctfg it is not enough to know that its Sylow subgroups are ctfg. DeMeyer and Janusz do provide enough additional conditions to assure that G is a ctfg.