

THE STRUCTURE OF TWISTED $SU(3)$ GROUPS

ALBERT JEU-LIANG SHEU

In order to study how the C^* -algebra $C(S_\mu U(3))$ of twisted $SU(3)$ groups introduced by Woronowicz is related to the deformation quantization of the Lie-Poisson $SU(3)$, we need to understand the algebraic structure of $C(S_\mu U(3))$ better. In this paper, we shall use Bragiel's result about the irreducible representations of $C(S_\mu U(3))$ and the theory of groupoid C^* -algebras to give an explicit description of the C^* -algebra structure of $C(S_\mu U(3))$, which indicates that $C(S_\mu U(3))$ is some kind of foliation C^* -algebra of the singular symplectic foliation of the Lie-Poisson group $SU(3)$.

In recent years, there has been a rapid growth of interest in the theory of quantum groups [D]. In particular, S. L. Woronowicz has developed a C^* -algebraic theory of quantum groups, which has motivated a lot of research [B, Po, Ro, S, Va-So, Wo1, Wo2].

In [S], the explicit knowledge of the C^* -algebra structure of $C(S_\mu U(2))$ [Wo1, S] has helped us to find a deformation quantization [BFFLS, Ri1, Ri2, Ri3] of the Lie-Poisson $SU(2)$ [D, Lu-We], which is in a sense compatible with the quantization of the group structure of $SU(2)$ by the "twisted groups" $S_\mu U(2)$. On the other hand, although both $C(S_\mu U(2))$ and $C(S_\mu U(3))$ [Wo1, Wo2] are defined as universal C^* -algebras of certain generators and relations, the algebraic structure of the latter seems to be much more complicated than that of the former. In [B], Bragiel classified the irreducible representations of the C^* -algebra $C(S_\mu U(3))$ of the twisted $SU(3)$ groups (with $0 < \mu < 1$) and showed that $C(S_\mu U(3))$ is a type-I C^* -algebra [Pe]. In this paper, enlightened by the ideas in [M-Re, Cu-M], we shall use Bragiel's result and the theory of groupoid C^* -algebras [Re] to give an explicit description of the C^* -algebra structure of $C(S_\mu U(3))$, which indicates that $C(S_\mu U(3))$ is some kind of foliation C^* -algebra of the singular symplectic foliation of the Lie-Poisson group $SU(3)$ [Co, We, Lu-We].

We shall use freely the concepts and properties of the theory of groupoid C^* -algebras throughout this paper. A good reference for this is [Re]. First let us fix notations. Let $\mathbb{T}$ be the unit circle in $\mathbb{C}$ and $\mathbb{T}^2$ be the two-torus embedded in $\mathbb{C}^2$. We shall denote by ϕ and