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LÉVY-HINČIN TYPE THEOREMS FOR MULTIPLICATIVE AND ADDITIVE FREE CONVOLUTION

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We give a description of infinitely divisible compactly supported probability measures relative to the multiplicative free convolutions on the positive half-line and on the unit circle. A new proof is provided for the analogous result for additive free convolution.

1. Introduction. In classical probability theory an important role is played by the convolution of probability distributions on the real line; indeed, the distribution of the sum of two independent random variables is the convolution of the distributions of the two summands. In connection with this, the infinitely divisible probability distributions, and the associated convolution semigroups, occupy a central place. On the one hand, infinitely divisible probability distributions are the natural object in the study of the limits of sums of independent random variables, and on the other, convolution semigroups are related with stationary processes with independent increments; see [**2**, **3**, and **4**] for further information in the classical situation. The Lévy-Hinčin formula gives a complete description of all the infinitely divisible distributions on the real line, and it shows that such distributions can be obtained as limits of convolutions of Gaussian and Poisson distributions.

In this paper we study infinite divisibility for multiplicative and additive free convolution, which are two operations arising from the non-commutative probability theory of free products; see [**5**, **6**, **7**, **8**, and **9**] for the background of this non-commutative theory. We will describe briefly the definition of the free convolution operations. Consider a unital algebra A, endowed with a functional ϕ such that $\phi(1) = 1$; the elements $x \in A$ will be called random variables. One can associate with each random variable x an analytic functional ν_x, i.e., a functional on the polynomial algebra $\mathbf{C}[X]$, by the formula $\nu_x(p) = \phi(p(x))$ if $p \in \mathbf{C}[X]$. If x and y are two free random variables (in a technical sense which we will not explain here) then ν_{x+y} and ν_{xy} can be shown to depend only on ν_x and ν_y. This allows one to define the additive and multiplicative free convolution