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ESTIMATING NIELSEN NUMBERS ON INFRASOLVMANIFOLDS

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A well-known lower bound for the number of fixed points of a self-map $f: X \to X$ is the Nielsen number $N(f)$. Unfortunately, the Nielsen number is difficult to calculate. The Lefschetz number $L(f)$, on the other hand, is readily computable, but does not give a lower bound for the number of fixed points. In this paper, we investigate conditions on the space X which guarantee either $N(f) = |L(f)|$ or $N(f) \geq |L(f)|$. By considering the Nielsen and Lefschetz coincidence numbers, we show that $N(f) \geq |L(f)|$ for all self-maps on compact infrasolvmanifolds (aspherical manifolds whose fundamental group has a normal solvable subgroup of finite index). Moreover, for infranilmanifolds, there is a Lefschetz number formula which computes $N(f)$.

1. Estimating Nielsen numbers. Consider a continuous self-map $f: X \to X$. Let $\mathrm{Fix}(f)$ denote the fixed point set $\{x \in X | f(x) = x\}$. One of the fundamental problems of fixed point theory is to estimate (preferably from below) the cardinality of this set. The Nielsen number $N(f)$ provides such an estimate: it is an integer homotopy invariant which provides a lower bound on the number of fixed points of g, for all maps g homotopic to f. This estimate is sharp for all compact manifolds save surfaces of negative Euler characteristic. Its one drawback is that it is very difficult to compute $N(f)$ from its definition, so that other means must be sought. At least, since the Nielsen number provides a lower bound for the original topological object $|\mathrm{Fix}(f)|$, it would be useful to find lower bounds for $N(f)$. We will refer to the search for lower bounds to $N(f)$ as the problem of *estimating* $N(f)$; while the search for other algebraic-topological means of finding the exact value of $N(f)$ will be referred to as the problem of *computing* $N(f)$.

The Lefschetz number $L(f)$ is a (reasonably) computable invariant, but in general, there is no relation between $L(f)$ and either $N(f)$ or $|\mathrm{Fix}(f)|$. One approach to computing the Nielsen number is to find conditions on either the space X or the map f which allow $N(f)$ and $L(f)$ to be related. The Jiang condition, for example, is a condition on the map f which, when satisfied, computes $N(f)$ from $L(f)$