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ORIENTATION AND STRING STRUCTURES ON LOOP SPACE

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This paper deals rigorously with the notion of a string structure and the topological obstruction to its existence. The question of orienting loop space is discussed and shown to be directly analogous to orientation of finite dimensional manifolds. Finally, equivariant string structures are considered.

1. Introduction. Physicists working on the grand unification program have recently been led to consider particles, not as points on some manifold M, but rather as loops on M. This novel idea has resulted in efforts to formulate a theory of spinors on LM, the free loop space of M [**3**]. The theory is called string theory.

Since LM is an infinite dimensional manifold, placing this on a rigorous mathematical footing is a real challenge. The first problem is to define the Dirac operator and involves constructing a spinor bundle on which it acts. E. Witten and T. P. Killingback have made considerable progress in this regard.

Witten in [**19**], [**20**] and Atiyah in [**2**] argued that LM should be considered orientable exactly when M is a spin manifold. Killingback [**10**] looked at bundles on LM whose structural groups are loop groups. He defined a string structure as a lifting of the structural group to a central extension of the loop group by a circle. The candidate for the spinor bundle on LM is then a certain infinite dimensional vector bundle associated with the string structure. Just as in finite dimensions, there is a topological obstruction to defining this bundle. Killingback argued that it is essentially the first Pontrjagin class of M. In this paper, we clarify these results and prove them rigorously.

In §2, we examine the orientability of loop space. Suppose that $P \to M$ is an $\mathrm{SO}(n)$-bundle. By taking free loops, we obtain an $L\,\mathrm{SO}(n)$-bundle $LP \to LM$ in a natural way. Assuming that M is simply connected, we show that it is possible to reduce the structural group of $LP \to LM$ to the connected component of the identity if and only if $P \to M$ admits a spin structure. The condition that M be simply connected is reasonable, since it is equivalent to LM