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QUADRATIC CENTRAL POLYNOMIALS WITH DERIVATION AND INVOLUTION

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The main result of this paper shows that if R is a prime ring with involution and with derivation D, then if $p(x, y) = c_1xy^D + c_2x^Dy + c_3yx^D + c_4y^Dx$ is central for all (skew-) symmetric elements of R, then R must embed in $M_2(F)$, with two explicit exceptions. As a consequence of the special case when $x = y$, one obtains generalizations of existing results about (skew-) centralizing derivations of the (skew-) symmetric elements.

Introduction. The motivation for this paper lies in an attempt to classify the minimal homogeneous identities with derivation which hold for the (skew-) symmetric elements in an ideal of a prime ring R with involution. As a consequence of **[11]**, there are two specific types of such identities $f(x^d, y^h)$ of degree two if R does not satisfy a polynomial identity and $\operatorname{char} R \neq 2$, and no such identity can be of the form $f(x, y^h)$. The situation when either $\operatorname{char} R = 2$ or R is a PI ring, and about other degree two homogeneous identities not of these forms, remains to be studied. In this paper we investigate identities of the form $p(x, y) = c_1xy^D + c_2x^Dy + c_3yx^D + c_4y^Dx$, and more generally show that $p(x, y)$ cannot be a central polynomial for the (skew-) symmetric elements except in two specific cases, or when R embeds in $M_2(F)$. The results in **[11]** are not applicable here since now the same variable appears both with and without a derivation applied.

Throughout the paper, R will denote a prime ring with center Z, extended centroid C, and Martindale quotient ring Q **[15]**. Henceforth, we shall assume that R has an involution, *, and for any ideal I of R we set $T(I) = \{r + r^* | r \in I\}$, $S(I) = \{r \in I | r^* = r\}$, and $K(I) = \{r - r^* | r \in I\}$. It is easy to to show that * extends to C **[16]**. We say that * is of the first kind if $C = C \cap S$, and is of the second kind otherwise. In general, the latter case is easy to deal with. For D a nonzero derivation of R, it is easy to check that D extends uniquely to a derivation of Q, so restricts to a derivation of the central closure $RC + C$ of R (see **[8]**). We say that D is inner if its extension to Q is the inner derivation $\operatorname{ad}(A)(x) = xA - Ax$, for $A \in Q$, and otherwise call D outer.