

UNIVERSAL CLASSES OF ORLICZ FUNCTION SPACES

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It is shown that for each $0 < p < q < \infty$ the space $L^p(0, \infty) + L^q(0, \infty)$, defined as in Interpolation Theory, is universal for the class of all Orlicz function spaces L^ψ with Boyd indices strictly between p and q (i.e. every Orlicz function space L^ψ is order-isomorphically embedded into $L^p(0, \infty) + L^q(0, \infty)$). The extreme case of spaces having Boyd indices equal to p or q is also studied. In particular every space $L^r(0, \infty) + L^s(0, \infty)$ embeds isomorphically into the sum $L^p(0, \infty) + L^q(0, \infty)$ for any $0 < p \leq r \leq s < q < \infty$.

0. Introduction. It is a well-known fact from Interpolation Theory (cf. [L-T II] Proposition 2.b.3) that given $1 \leq p < q \leq \infty$, every rearrangement invariant (r.i.) Banach function space X on the interval $(0, \infty)$ with Boyd indices between p and q , is an intermediate space between the spaces $L^p(0, \infty) \cap L^q(0, \infty)$ and $L^p(0, \infty) + L^q(0, \infty)$. This means that $L^p(0, \infty) \cap L^q(0, \infty) \subset X \subset L^p(0, \infty) + L^q(0, \infty)$ with continuous inclusions.

One of the purposes of this paper is to study the universality of the spaces $L^p(0, \infty) + L^q(0, \infty)$ with respect to classes of intermediate r.i. function spaces X , in the sense of whether the above inclusion $X \subset L^p(0, \infty) + L^q(0, \infty)$ can be replaced by suitable isomorphic embeddings of X into $L^p(0, \infty) + L^q(0, \infty)$. At the same time, these spaces $L^p(0, \infty) + L^q(0, \infty)$ can be regarded as Orlicz function spaces $L^\varphi(0, \infty)$, taking the Orlicz function $\varphi(x) = x^p \wedge x^q = \min(x^p, x^q)$, and, consequently, we are also interested in finding universal Orlicz function spaces for classes of quasi-Banach Orlicz function spaces $L^\psi(0, \infty)$.

These questions are motivated in part by the existence of positive results of universality in the context of Orlicz sequence spaces: J. Lindenstrauss and L. Tzafriri (cf. [L-T I] Theorem 4.b.12) proved that there exist universal Orlicz sequence spaces l^φ with arbitrary prefixed indices $1 \leq p < q < \infty$ such that every Orlicz sequence space l^ψ with indices between p and q is isomorphic to a (complemented) subspace of l^φ .

We show here that, in general, for $0 < p < q < \infty$, the spaces $L^p(0, \infty) + L^q(0, \infty)$ are universal for the class of all Orlicz function