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MINIMAL ORBITS AT INFINITY IN HOMOGENEOUS SPACES OF NONPOSITIVE CURVATURE

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Let M denote a simply connected, homogeneous space of nonpositive curvature and let G be the connected component of the identity of the isometry group of M.

In this paper we study the geometric consequences on M if $M(\infty)$, the boundary sphere of M, admits a G-orbit whose closure is a minimal set for G. A characterization of symmetric spaces of noncompact type in terms of the action of G in $M(\infty)$, is obtained. As an application we give some conditions, in terms of the Lie algebra of a simply transitive and solvable subgroup of G that is in standard position, which are equivalent to the fact that M is a symmetric space.

Introduction. Let M denote a simply connected, homogeneous space of nonpositive curvature $(K \leq 0)$ and let G be the connected component of the identity in $I(M)$, the isometry group of M.

In this paper we study the geometric consequences on M if $M(\infty)$, the boundary sphere of M, admits a G-orbit whose closure is a minimal set for G. In particular, we obtain a characterization of symmetric spaces of noncompact type in terms of the action of G in $M(\infty)$. As an application, some conditions in terms of properties of the Lie algebra of a simply transitive, solvable subgroup of G that is in standard position, which are equivalent to the fact that M is a symmetric space, are obtained.

In §1 we give a characterization of symmetric spaces in terms of the G-minimality of the closure of some orbits of G in $M(\infty)$, or equivalently in terms of K, the stability subgroup of G at any point in M, we obtain that M is a symmetric space of noncompact type if and only if $G(x) = K(x)$ for a particular x in $M(\infty)$ (Theorem 1).

In §2 we get a decomposition of $\mathscr{G}$, the Lie algebra of G, that coincides with the canonical one when M is symmetric. It is used to give, as an application of Theorem 1, a characterization of symmetric spaces of noncompact type in terms of properties of the Lie algebra of a simply transitive, solvable group of isometries of M that is in standard position (Theorem 2).