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PSEUDO REGULAR ELEMENTS AND THE AUXILIARY MULTIPLICATION THEY INDUCE

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An element f of a commutative Banach algebra is *pseudo regular* if there is a constant M with $\|abf\| \leq M\|af\|\,\|bf\|$ $(a, b \in \mathfrak{A})$. In many cases pseudo regularity implies formally stronger conditions such as relative invertibility; that is, f is invertible in some subalgebra of $\mathfrak{A}$. In this paper we describe some algebraic methods which can be used to establish results of this kind. Given a pseudo regular element f of $\mathfrak{A}$, $af \circ bf = abf$ extends by continuity to a multiplication $\circ$, called the *auxiliary multiplication*, on J, the closed ideal generated by f. This leads to the fundamental inequality $\|\phi\|_{J^*} \leq M|\phi(f)|$ where ϕ is a multiplicative linear functional on $\mathfrak{A}$. As applications of these ideas we identify the pseudo regular elements of the algebra $C^{(n)}[0, 1]$ as being the elements such that $f, f', \dots, f^{(n)}$ have no common zeros and the pseudo regular elements of the group algebra of a locally compact abelian group as being the relatively invertible elements. Similar constructions can be made when f is an element of an $\mathfrak{A}$ module $\mathfrak{X}$ though the structure is less rich in this case.

1. Introduction. The idea of pseudo regularity was introduced by Arens in [**1**] where he first proved results of the kind in this paper. The definition of auxiliary multiplication is given in §2. The product $j \circ k$ is, heuristically, jk/f and this is made precise in Proposition 2.4 and Corollary 2.5. The fundamental inequality is proved in §3 where the equivalence of pseudo regularity and relative invertibility also appears (Corollary 3.6). The two remaining sections deal with pseudo regularity of an element f of an $\mathfrak{A}$ module $\mathfrak{X}$ (§4) and of a system (§5). In §5 we show that a system $F = (f_1, \dots, f_n) \in \mathfrak{A}^n$ where $\mathfrak{A}$ is a uniform algebra is pseudo regular if and only if $(0, \dots, 0)$ is an isolated point of $\{(\phi(f_1), \dots, \phi(f_n)),\ \phi \in \partial\mathfrak{A}\} \cup \{(0, \dots, 0)\}$ where $\partial\mathfrak{A}$ is the Šilov boundary of $\mathfrak{A}$. This extends a result in [**1**] for the case $n = 1$.

The author is indebted to Professor Arens for letting him have a copy of an early version of [**1**].

2. The auxiliary multiplication generated by a pseudo regular element. Let $\mathfrak{A}$ be a commutative Banach algebra and let $f \in \mathfrak{A}$. We