PACIFIC JOURNAL OF MATHEMATICS
Vol. 161, No. 1, 1993

THE RELATIVE NIELSEN NUMBER AND BOUNDARY-PRESERVING SURFACE MAPS

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Let X be a compact 2-manifold with nonempty boundary ∂X. Given a boundary-preserving map $f: (X, \partial X) \to (X, \partial X)$ the relative Nielsen number $N_\partial(f)$ gives a lower bound for the number of fixed points of f. Let $MF_\partial[f]$ denote the minimum number of fixed points of all boundary-preserving maps homotopic to f as maps of pairs. This paper continues the study of the difference $MF_\partial[f] - N_\partial(f)$ for surface maps begun by Brown and Sanderson [BS]. Their results are extended by (i) adding to their list of surfaces for which this difference can be arbitrarily large, and (ii) producing an example of a boundary-preserving map of the pants surface for which the difference is equal to one. This answers a conjecture raised by the authors.

1. Introduction. Let X be a compact, connected surface and let ∂X denote its boundary. Given a self-map $f: X \to X$, following standard notation, let $N(f)$ denote the Nielsen number of f and let $MF[f]$ be the minimum number of fixed points possible for a map homotopic to f. It is a classical result that for arbitrary spaces and maps the difference $MF[f] - N(f)$ is always a non-negative integer. For manifolds of dimension greater than two, it is a well-known result of Wecken [**W**] that $MF[f] = N(f)$. While on the other hand, Jiang [**J**] showed that any surface with negative Euler characteristic supports maps for which the difference is strictly positive. Results of the author ([**K1**], [**K2**]) have established that for any surface which contains a cohomology injective embedding of the pants surface, the difference can be made arbitrarily large.

In the paper [**BS**] the authors begin a study of the fixed point behavior for boundary-preserving maps of surfaces. (These are relative maps $f: (X, \partial X) \to (X, \partial X)$.) It is assumed that all homotopies under consideration are homotopies through boundary-preserving maps. In this setting, the relative Nielsen number as defined by Schirmer [S], and denoted $N_\partial(f)$, is a better algebraic invariant than the standard Nielsen number. The analogous minimal number to consider will be denoted $MF_\partial[f]$. The intent of their work is to classify all surfaces X (with $\partial X \neq \varnothing$) in terms of the following: