PACIFIC JOURNAL OF MATHEMATICS
Vol. 161, No. 2, 1993

REFLEXIVITY OF SUBNORMAL OPERATORS

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Dedicated to Donald Sarason, in admiration of the range of his pioneering work

We give a new proof that subnormal operators are reflexive. We extend this to certain subnormal n-tuples. We give the first complete proof that a pair of doubly commuting isometries is reflexive.

0. Introduction. Let A be a weakly closed algebra of bounded linear operators on a Hilbert space $\mathscr{H}$. Its lattice, $\mathrm{Lat}(A)$, is the set of all closed subspaces of $\mathscr{H}$ that are left invariant by every element of A. The set of operators that leave invariant every space in $\mathrm{Lat}(A)$ is denoted $\mathrm{Alg}\,\mathrm{Lat}(A)$. The algebra A is called *reflexive* if $A = \mathrm{Alg}\,\mathrm{Lat}(A)$. An operator (or set of operators) T is called reflexive if the weakly closed unital algebra it generates, $W(T)$, is reflexive.

D. Sarason proved that normal operators are reflexive, and that so are analytic Toeplitz operators **[Sa1]**. R. Olin and J. Thomson extended this result in 1979 to prove that all subnormal operators (i.e. restrictions of normal operators to invariant subspaces) are reflexive **[OT]**. Whilst their original proof has been somewhat simplified since then **[Th1]**, **[Th2]**, **[Co1]**, to date all proofs have relied on an elaborate construction of "full analytic subspaces". We show that Thomson's work on bounded point evaluations **[Th3]** allows a much simpler proof (Theorem 1).

An n-tuple of operators $N = (N_1, \dots, N_n)$ is called *normal* if each N_i is normal, and $N_iN_j = N_jN_i$ for all i, j. The n-tuple $S = (S_1, \dots, S_n)$ of operators on $\mathscr{H}$ is called *subnormal* if there is a Hilbert space $\mathscr{K}$ containing $\mathscr{H}$, and a normal n-tuple $(N_1, \dots, N_n)$ on $\mathscr{K}$, such that each N_i leaves $\mathscr{H}$ invariant, and $N_i|_{\mathscr{H}} = S_i$. Just as in the cyclic case, a subnormal n-tuple is reflexive if its restriction to every cyclic subspace is. Moreover, any cyclic subnormal n-tuple is unitarily equivalent to $(M_{z_1}, \dots, M_{z_n})$ on $P^2(\mu)$ for some compactly supported measure μ on $\mathbb{C}^n$ (see §2). So studying reflexivity reduces to studying the spaces $P^2(\mu)$ and the n-tuple S_μ of multiplication by the variables.

A point λ in $\mathbb{C}$ is called a *bounded point evaluation* for $P^2(\mu)$ if the functional of evaluating at λ, defined on the polynomials, is bounded