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CONFORMAL REPELLORS WITH DIMENSION ONE ARE JORDAN CURVES

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We show that a conformal repellor in $\mathbb{R}^m$ whose Hausdorff and topological dimensions are equal to 1 is a Jordan curve. Moreover, its 1-dimensional Hausdorff measure is finite and it has a tangent at every point.

Introduction. In this note we study the topological and metric structure of conformal repellor $X \subset \mathbb{R}^m$, $m \geq 1$, of topological dimension 1. The definition of conformal repellor is given in the next two sections. We then show the following dichotomy: either the Hausdorff dimension of X exceeds 1 or else X is a Jordan curve (simple closed curve) and its 1-dimensional Hausdorff measure is positive and finite. Moreover, in the latter case X has a tangent at every point - X is smooth. This result generalizes Lemma 3 of [**PUZ**] which is formulated in the plane case ($m = 2$). The proof contained in [**PUZ**] uses the Riemann mapping theorem and can be carried out only in the plane. The proof presented in our paper is different and holds in any dimension. The reader is also encouraged to notice an analogy between our result and a series of other recent papers (see for examples [**B, P, R1, S, U, Z1, Z2**]) which are aimed toward establishing a similar dichotomy. However, to our knowledge, all these results were formulated in the plane case and have as a starting point the assumption that X is a continuum. Then the dichotomy is only that either the Hausdorff dimension of X exceeds 1 or X is a smooth curve.

THE SETTING. Let X be a nonempty compact subset of $\mathbb{R}^m$, U an open set, $X \subset U$ and f a map of U into $\mathbb{R}^m$ of class $C^{1+\alpha}, 0 < \alpha$, such that

(i) $f(X) \subset X$