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THE NONHOMOGENEOUS MINIMAL SURFACE EQUATION INVOLVING A MEASURE

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We find existence of a minimum in BV for the variational problem associated with $\operatorname{div} A(Du) + \mu = 0$, where A is a mean curvature type operator and μ a nonnegative measure satisfying a suitable growth condition. We then show a local L^∞ estimate for the minimum. A similar local L^∞ estimate is shown for sub-solutions that are Sobolev rather than BV.

1. Introduction. In this paper we initiate an investigation of weak solutions of the

$$\operatorname{div} A(Du) + \mu = 0 \tag{1.1}$$

in a bounded Lipschitz domain $\Omega \subset R^n$. Here A is a function for which the mean curvature operator is a prototype and μ is a nonnegative Radon measure supported in Ω that satisfies

$$\mu(B(r)) \leq Mr^{q(n-1)} \text{ for all } B(r) \subset \Omega, \tag{1.2}$$

where $M > 0$ and $1 < q \leq \frac{n}{n-1}$.

This paper has its origins in the work of [**LS**] where it was shown that if u is a weak solution of

$$\Delta u = \mu,$$

where μ is a measure that satisfies the growth condition

$$\mu(B(r)) \leq Mr^{n-2+\varepsilon}$$

for some $\varepsilon > 0$ and for all balls $B(r)$ of radius r, then u is Hölder continuous. In [**RZ**] this result was generalized to equations of the form

$$\operatorname{div} A(x, u, \nabla u) + B(x, u, \nabla u) + \mu = 0 \tag{1.3}$$