PACIFIC JOURNAL OF MATHEMATICS
Vol. 167, No. 1, 1995

NON-COMPACT TOTALLY PERIPHERAL 3-MANIFOLDS

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A 3-manifold is totally peripheral if every loop is freely homotopic into the boundary. It is shown that an orientable 3-manifold M is totally peripheral if and only if there is a boundary component F of M such that the inclusion of F in M induces a surjective map of fundamental groups. If M is non-orientable, there are essentially two counterexamples.

A 3-manifold is *totally peripheral* if every loop is freely homotopic into the boundary. Brin, Johannson and Scott studied compact totally peripheral 3-manifolds. They showed that when M is orientable, compact and totally peripheral, then there is a boundary component F of M such that the natural map $\pi_1(F) \to \pi_1(M)$ is surjective. When M is non-orientable, they showed that this result is almost true but that there are essentially two counterexamples. In this paper, we show that the same results hold if the compactness hypothesis on M is omitted. The results remain true even if the fundamental group of M is not finitely generated.

Brin, Johannson and Scott [**1**] also proved a relative version of their results. We say that a 3-manifold is totally peripheral relative to a subsurface B of ∂M (possibly B is disconnected), or TP rel B, if every loop in M is freely homotopic into B. They showed that if M is orientable, compact and totally peripheral relative to a compact subsurface B of ∂M, then there is a component C of B such that the natural map $\pi_1(C) \to \pi_1(M)$ is surjective. This relative result is also a consequence of our result for the non-compact case as, given a compact manifold M and a compact subsurface B in ∂M such that M is TP rel B, one can remove the closure of ∂M - B from M to obtain a non-compact totally peripheral 3-manifold M' with boundary equal to the interior of B. However, we use the relative case of [**1**] in the proof of our results.