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DESINGULARIZATIONS OF SOME UNSTABLE ORBIT CLOSURES

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Let σ be a semisimple automorphism of a connected reductive group G, and let G_σ be the fixed points of σ. We consider the G_σ-orbits on the space of nilpotent elements in an eigenspace of $d\sigma$. We give a desingularization of the orbit closures and relate the G_σ-orbits to the G-orbits. Along the way, we describe the fixed points of σ on a flag variety G/P where P is a σ-stable parabolic subgroup of G.

I. Introduction. In this note we observe some consequences of Richardson's theorems on orbits of reductive groups, in the following situation. Let G be a simply-connected reductive algebraic group over an algebraically closed field F whose characteristic is either zero or sufficiently large (as specified below). Let $\mathfrak{g}$ be the Lie algebra of G, and let $\mathcal{N}$ be the variety of nilpotent elements in $\mathfrak{g}$. Let σ be a semisimple automorphism of G, fix a nonzero element $q \in F^\times$, and consider the variety

$$\mathcal{N}_{\sigma,q} = \{x \in \mathcal{N} : \ d\sigma(x) = qx\}.$$

If q is not a root of unity then $\mathcal{N}_{\sigma,q}$ is the whole q-eigenspace of $d\sigma$, hence is a linear subspace of $\mathfrak{g}$. If q is a root of unity, the variety $\mathcal{N}_{\sigma,q}$ may even be reducible.

It was shown by Steinberg that the group of σ-fixed points G_σ is also a connected reductive F-group ([**S**]). The adjoint action of G_σ preserves each eigenspace of $d\sigma$, and $\mathcal{N}_{\sigma,q}$ consists of those G_σ-orbits in the q-eigenspace of $d\sigma$ which are "unstable", in the sense of geometric invariant theory ([**H2**]). According to a theorem of Kac and Richardson ([**Ri3**]), the G_σ-orbits on $\mathcal{N}_{\sigma,q}$ are exactly the irreducible components of sets of the form $\mathcal{N}_{\sigma,q} \cap \tilde{\mathcal{O}}$, where $\tilde{\mathcal{O}}$ is a nilpotent G orbit. Richardson also proved (with our assumptions on the characteristic of F, see [**Ri1**]) that there are only finitely many